2 Motion in a Straight Line

Any game, when played professionally, is broken down to basics and studied in depth. When a sprinter takes off, his acceleration, stride and time is noted to increase his performance. A cricketer studies the concept of impulse to strike the ball for perfect timing. All fields of sports require an in-depth knowledge of kinematics. Kinematics is a branch of mechanics which deals with the study of motion on a body. Hence for all the 2 and 3-dimensional motion, study of kinematics is important.

Topic Notes

- *Mechanics*

MECHANICS 1

Statics: It is the branch of mechanics that deals with the study of physical bodies at rest.

Kinematics: It is the branch of mechanics which deals with the study of the motion of physical bodies in two dimensions without taking into account the factors which cause motion.

Dynamics: It is the branch of mechanics which deals with the study of motion with acceleration of physical bodies in three dimensions with taking into account the factors which cause motion.

Frame of Reference: Motion of a body can be observed only if it changes its position with respect to some other body. Therefore, for motion to be observed, there must be a body which is changing its position with respect to another body and a person who is observing motion. The person observing motion is known as the observer. The observer for the purpose of investigation must have its own clocks to measure time and a point in the space attached with the other body as origin and a set of coordinate axes, these are collectively called a reference frame. Thus, frame of reference is a system of coordinate axes attached to an observer having a clock with him, with respect to which, the observer can describe position, displacement, acceleration etc. of a moving object.

TOPIC 1

MOTION AND REST

Motion

An object is said to be in motion if it changes its position with respect to its surroundings. For example, when we walk, run or ride a bike we are in motion with respect to the ground. Motion can be of various types: rectilinear motion, circular motion, oscillatory motion and one-dimensional, two-dimensional and three-dimensional motion.

Rest

An object is said to be at rest if it doesn't change its position with respect to the surroundings. For example, the whiteboard in the classroom is at rest with respect to the classroom.

Motion/rest is always relative to the observer. Motion/rest is a combined property of the object under study and the observer. There is no meaning of rest or motion without the observer or Frame of reference. To locate the position of a particle, we need a reference frame. The most commonly used reference frame is a *Cartesian Coordinate System* or $x - y - z$ coordinate system. The coordinates (x, y, z) of the particle specifies the position of the particle with respect to the origin of that frame:

(1) If all the three coordinates of the particle remain unchanged as time passes, it means the particle is at rest with respect to this frame.

(2) If only one coordinate changes with time, motion is one-dimensional (1-D) or straight-line motion.

(3) If only two coordinate changes with time, motion is two-dimensional (2-D) or motion in a plane.

(4) If all three coordinates change with time, motion is three-dimensional (3-D) or motion in a space.

If the frame is not mentioned, then the ground is taken as a reference frame.

Scalar and Vector Quantities

The physical quantities which have only magnitude but no direction, are called scalar quantities.

The physical quantities which have magnitude as well as direction, are called vector quantities. Vectors cannot be added, subtracted and multiplied. A vector is represented by a single letter with an arrow on its head.

Example 1.1: Can a body exist in a state of absolute rest or in absolute motion? Explain.

Ans. Absolute rest and motion are unknown, considering a point absolutely fixed in space has to be chosen as a reference point, but there is no such point in the universe. The earth revolves around the sun. The entire solar system travels through our own galaxy, the Milky Way, and clusters of galaxies now with respect to the other clusters. Thus, no object in the universe is in a state of absolute rest, so the absolute motion cannot be realised, only relative rest and motion can be realised.

Example 1.2: For the given examples of motion, in each case the motion is one, two, or three-dimensional?

(A) A kite flying on a windy day.

(B) A speeding car on a long straight highway.

(C) An insect crawling on a globe.

(D) **A carrom coin rebounding from the side of the board.**

(E) **A planet revolving around its star.**

Ans. (A) The motion of the kite in the sky is three-dimensional motion because the kite while flying may be changing the direction as well as losing or gaining height, *i.e.*, motion is in space.

(B) The motion of a speeding car on a long straight highway is one-dimensional motion since the motion is confined along a straight line.

(C) The motion of crawling of an insect on a globe is three-dimensional because it has three dimensions as it has a spherical shape.

(D) The motion of a carrom coin rebounding the side of the board is a two-dimensional motion, since the motion is confined in the plane of the board.

(E) The motion of a planet revolving around its star is two-dimensional motion because the motion is confined in a fixed plane.

TOPIC 2

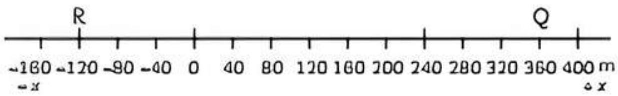
MOTION IN A STRAIGHT LINE

Point Object

If the position of an object changes by distance, much greater than its size in a reasonable duration of time, then the object may be regarded as a point object; when a point object moves, its rotational and vibrational motion may be ignored. For example, earth can be regarded as a point object for studying its motion around the sun or a train under a journey of several hundred kilometers can be regarded as a point object.

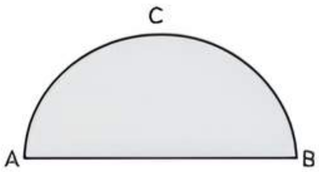
Position

It is defined as the point where an object is situated and is always defined with respect to some reference point which we generally refer to as origin. For example, to describe motion along a straight line, choosing an X-axis, so that it coincides with the path of the object. Now measuring the position of the object with reference origin O, positions to the right of O are taken as positive and to the left of O, as negative. Following this convention, the position coordinates of point P and Q in figure are +360 m and +240 m. Similarly, the position coordinate of point R is –120 m.



Path Length or Distance

Distance covered by the object in a given time is called path length. Let a body moving from A to B is equal to C. The length of path ACB is called the distance travelled by the body.



But the overall body is displaced from A to B. A vector from A to B *i.e.*, its displacement vector or displacement that is the minimum distance and directed from initial.

Displacement

The change in position of an object in a particular direction is termed as displacement, *i.e.*, the difference between the final and initial position of the object in a given time. It is denoted by Δx.

Mathematically, it is represented by,

$$\Delta x = x_2 - x_1$$

where, x_1 and x_2 are the initial and final positions of the object, respectively.

Cases:

(1) If $x_2 > x_1$, then it is positive.

(2) If $x_1 > x_2$, then it is negative.

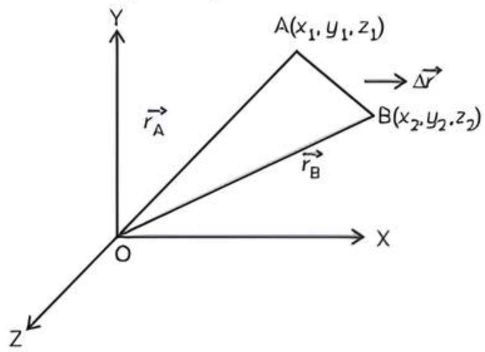
(3) If $x_1 = x_2$, then it is zero.

Important

➥ *Distance is a scalar, while displacement is a vector.*

Distance depends on our path, while displacement is independent of the path but depends only on final and initial positions.

Let a body be displaced from A to B, then its displacement is given by $\overrightarrow{AB}$.



From, $\vec{r}_A + \Delta\vec{r} = \vec{r}_B$ or $\Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \text{ and}$$

$$\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

or $$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

Example 1.3: Is the magnitude of the displacement of an object and the total distance covered by it in certain time intervals the same? Explain.

Ans. According to the properties of displacement, the displacement of an object in a given time interval can be positive, zero, or negative and the actual distance travelled by an object in a given time interval is either equal to or greater than the magnitude of displacement, thus it cannot be the same.

Example 1.4: A particle starts from the origin goes along the X-axis upto the point (20 m, 0) and then returns along the same line to the point (–20 m, 0). Find the distance and displacement of the particle during the trip.

Ans.

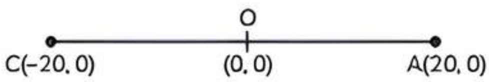
$$\text{Distance} = |OA| + |AC| = 20\hat{i} + (+40\hat{i})$$

$$= (60\hat{i})\text{m}$$

$$\text{Displacement} = OA + AC$$

$$= 20\hat{i} + (-40)\hat{i}$$

$$= -(20\hat{i})\text{m}$$



Example 1.5: In which of the following examples of motion object can be considered approximately a point object?

(A) A railway carriage moving without jerks between two stations.

(B) A monkey sitting on top of a man cycling smoothly on a circular track.

(C) A spinning cricket ball that turns sharply on hitting the ground.

(D) A tumbling beaker that has slipped off the edge of a table.

Ans. **(A)** The size of the carriage is very small as compared to the distance between the two stations. Therefore, the carriage can be treated as a point-sized object.

(B) The size of a monkey is very small as compared to the size of a circular track. Therefore, the monkey can be considered as a point-sized object on the tracks.

(C) The size of the spinning cricket ball is comparable to the distance through which it turns sharply on hitting the ground. Hence, the cricket ball cannot be considered as a point object.

(D) The size of a beaker is comparable to the height of the table from which it slipped. Hence, the beaker cannot be considered as a point object.

Speed

The path length or the distance covered by an object and the time taken to cover that distance is called speed.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Its SI unit is meter per second (ms^{-1}) and dimension is $[M^0 L T^{-1}]$.

Uniform Speed

When an object covers equal distances in equal intervals of time (no matter how small the interval are), an object is said to be in uniform speed.

$$\text{Uniform Speed} = \frac{\text{Distance}}{\text{Time}}$$

In non-uniform variable speed, object covers unequal distances in equal intervals of time.

Average Speed

The average speed of a particle for a given interval of time is defined as the ratio of distance travelled to the time taken.

$$\text{Average speed} = \frac{\text{Total Distance Travelled}}{\text{Time taken}}$$

i.e., $$v_{av} = \frac{\Delta s}{\Delta t}$$

Instantaneous Speed

The speed of a particle at a particular instant of time.

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Important

➥ *When a particle moves with different uniform speeds in different time intervals respectively, its average speed over the total time of journey is given as:*

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{s_1 + s_2 + s_3 + \ldots + s_n}{t_1 + t_2 + t_3 + \ldots + t_n}$$

$$= \frac{u_1 t_1 + u_2 t_2 + u_3 t_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

If $t_1 + t_2 + t_3 + \ldots t_n = t_n$

Then

$$v_{av} = \frac{v_1 + v_2 + v_3 + \ldots + v_n}{n}$$

(Arithmetic mean of speeds)

Velocity

The rate of change of position *i.e.*, rate of displacement with time is called velocity. It's a vector quantity whose SI unit is m/s and CGS unit is cm/s. Its dimensions are $[LT^{-1}]$.

Uniform Velocity

A particle is said to have uniform velocity if the magnitude, as well as direction of its velocity, remain the same. This is possible only when it moves in a straight line without reversing its direction.

Non-Uniform Velocity

A particle is said to have non-uniform velocity if both either magnitude or direction of velocity changes.

Average Velocity

The ratio of displacement to the time taken by a body is defined as average velocity.

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time Taken}} = \frac{d}{t}$$

Its direction is along with the displacement.

Important

- *If velocity is changing with time, then time average velocity at instant time is given as:*

$$(v)_t = \frac{\int v\,dt}{\int dt}$$

and if it is changing with distance, then average velocity at a certain distance is given as

$$(v)_s = \frac{\int v\,ds}{\int ds}$$

Average speed ≥ |Average velocity|

Instantaneous Velocity

It is the velocity of a particle at a particular instant of time and given as:

$$v = \lim_{\Delta t \to 0} \frac{\delta s}{dt}$$

$$= \frac{ds}{dt}$$

When a particle has uniform velocity then its instantaneous speed, magnitude of instantaneous velocity, and average velocity all are equal.

TOPIC 3

ACCELERATION

The rate of change of velocity of an object is called the acceleration of the object. It is a vector quantity and its direction is the same as that of change in velocity. Its SI unit m/s^2, C.G.S unit is cm/s^2 and dimensions are $[M^0L^1T^{-2}]$.

Uniform Acceleration

A body is said to have uniform acceleration if the magnitude and direction of the acceleration remain constant during the motion of a particle.

Non-uniform Acceleration

A body is said to have non-uniform acceleration if either magnitude or direction or both changes during the motion of a particle.

Average Acceleration

It is the ratio of the total change in velocity and total time taken by the particle.

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

Instantaneous Acceleration

It is the acceleration of a particle at a particular instant of time.

$$a = \frac{\Delta v}{\Delta t} = \frac{d\vec{v}}{dt}$$

i.e., The first derivative of velocity is called Instantaneous acceleration.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \qquad \left[\text{As } \vec{v} = \frac{d\vec{r}}{dt}\right]$$

i.e., second derivative of the position vector is called instantaneous acceleration.

Important

- *When a particle moves with non-uniform speed, then acceleration of the particle must be non-zero. Acceleration which opposes the motion of the body, is called retardation.*
- *The sign of velocity (+ve or -ve) represents the direction of motion, but the sign of acceleration indicates the direction of acceleration.*

Important

➥ *If acceleration and velocity both are having the same sign then the magnitude of the velocity (i.e., speed) is increasing and if both have opposite signs then the magnitude of the velocity (i.e., speed) is decreasing.*

When the acceleration is uniform, obviously, instantaneous acceleration equals the average acceleration over that period. Therefore, Acceleration may result from a change in the speed(magnitude), a change in direction, or changes in both. Like velocity, acceleration can also be positive, negative, or zero.

Since velocity is a quantity having both magnitude and direction, a change in the velocity may involve either or both of these factors.

Example 1.6: A particle moves on a circular path of radius 5 m with constant velocity 5 m/s. Find the magnitude of the average acceleration when it completes a half revolution. [NCERT]

Ans. When the particle completes a half revolution, change in velocity,

$$\Delta v = [5-(-5)]\text{m/s} = 10 \text{ m/s}$$

Time taken to complete the half revolution is,

$$t = \frac{\pi r}{v} = \frac{\pi \times 5}{5} = \pi \text{ s}$$

Thus, average acceleration $= \frac{\Delta v}{t} = \frac{10}{\pi}\text{m/s}^2$

Example 1.7: The acceleration experienced by a boat after the engine is cut off is given by $\frac{dv}{dt} = -kv^3$, where k is a constant. If v_0 is the magnitude of the velocity at cut-off, find the magnitude of the velocity at time t after the cut-off. [NCERT]

Ans. The regarding acceleration $a = -kv^3$ which can be written as $\frac{dv}{dt} = -kv^3$ or $v^3 dv = -k$ at integrating both sides

$$\frac{-1}{2v_0^2} = -kt + C$$

$$C = \frac{-1}{2v_0^2}$$

So, the equation becomes

$$\frac{-1}{2v^2} = -kt + \frac{-1}{2v^2}$$

$$\frac{1}{2v^2} = kt - \frac{1}{2v_0^2}$$

or $$2v^2 = \frac{1}{kt + 1/2v_0^2}$$

or $$v = \frac{v_0}{\sqrt{1 + 2ktv_0^2}}$$

TOPIC 4

KINEMATIC EQUATIONS FOR UNIFORM ACCELERATED MOTION

If the change in velocity of an object in each unit time is constant, then the object is said to be moving with constant acceleration and such a motion is called uniform accelerated motion. An object moves along a straight line with a constant acceleration a and u be the initial velocity at $t = 0$ and v be the final value of the object after the time(t), then

Velocity time relation, $v = u + at$

Position time relation, $s = ut + \frac{1}{2}at^2$

Position velocity relation, $v^2 - u^2 = 2as$

Displacement of the object in the n^{th} seconds(n^{th})

$$s(n^{th}) = u + \frac{a}{2}(2n - 1)$$

Where a = acceleration = constant

u = initial velocity

v = final velocity

s = displacement

$s_{n^{th}}$ = Displacement in the n^{th} second

Non-uniform Accelerated Motion

When the acceleration of an object is not constant or acceleration is the function of time, then the following relation holds for one-dimensional motion.

$$v = \frac{dx}{dt}$$

$$dx = vdt$$

$$a = \frac{dx}{dt} \times \frac{dv}{dx}$$

$$dv = adt$$

or $$vdv = adx$$

All the equations of motion can be used in 2D motion in vector form.

Equation of Motion by Calculus Method

We know that:

(1) Acceleration is given as rate of change of velocity

$$a = \frac{dv}{dt}$$

Now on integrating both sides

$$a\int_0^t dt = \int_u^v dv$$

On solving the above we get

$$v = u + at$$

(2) Rate of the displacement is equal to velocity

$$v = \frac{ds}{dt}$$

On integrating the above equation we get

$$\int_0^a ds = \int_0^t v dt$$

On substituting the value;

$$v = u + at$$

We get,

$$s = ut + \frac{1}{2}at^2$$

(3) Distance travelled is equal to average speed time

$$s = \frac{u+v}{2}t$$

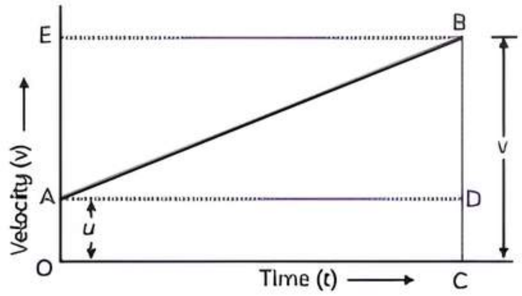
By using the first equation and substituting the value;

$$t = \frac{v-u}{a}$$

We get, $v^2 = u^2 + 2as$

Equation of Motion by Graphical Method

We know that:



In the above graph,

(1) The velocity of the body changes from A to B in time t at a uniform rate.

(2) BC is the final velocity and OC is the total time t.

(3) A perpendicular is drawn from B to OC, a parallel line is drawn from A to D, and another perpendicular is drawn from B to OE (represented by dotted lines).

The following details are obtained from the graph above:

The initial velocity of the body,

$$u = OA$$

The final velocity of the body,

$$v = BC$$

From the graph, we know that,

$$BC = BD + DC$$

Therefore, $v = BD + DC$

$v = BD + OA$ (since DC = OA)

Finally,

$v = BD + u$ (since OA = u) ...(i)

Now, since the slope of a velocity-time graph is equal to acceleration a.

So, a = slope of line AB

$$a = \frac{BD}{AD}$$

Since AD = AC = t, the above equation becomes:

$BD = at$ ...(ii)

Now, combining Equation (i) and (ii), the following is obtained:

$$v = u + at$$

From the given graph, we can say that

Distance travelled (s) = Area of figure OABC = Area of rectangle OADC + Area of triangle ABD

$$s = \left(\frac{1}{2} \times AD \times BD\right) + (OA \times OC)$$

As OA = u and OC = t,

the above equation becomes,

$$s = \left(\frac{1}{2} \times AD \times BD\right) + (u \times t)$$

As BD = at (from the graphical derivation of 1st equation of motion), the equation becomes,

$$s = \frac{1}{2} \times t \times at + ut$$

On further simplification, the equation becomes

$$s = ut + \frac{1}{2}at^2$$

From the graph, we can say that

The total distance travelled, s is given by the Area of trapezium OABC.

Hence, $s = \frac{1}{2} \times$ (Sum of Parallel Sides) × Height

$$s = \frac{1}{2} \times (OA + CB) \times OC$$

Since, OA = u,

CB = v,

and OC = t

Then the above equation becomes,

$$s = \frac{1}{2} \times (u + v) \times t$$

Now, $$t = \frac{(v-u)}{a}$$

The above equation can be written as:

$$s = \frac{1}{2} \times \frac{[(u+v) \times (v-u)]}{a}$$

Rearranging the equation, we get

$$s = \frac{1}{2} \times \frac{(v+u) \times (v-u)}{a}$$

$$s = \frac{(v^2 - u^2)}{2a}$$

Third equation of motion is obtained by solving the above equation:

$$v^2 = u^2 + 2as$$

Example 1.8: **A train, travelling at 20 km/hr is approaching a platform. A bird is sitting on a pole on the platform. When the train is at a distance of 2 km from the pole, brakes are applied which produce uniform deceleration in it. At that instant, the bird flies towards the train at 60 km/hr and after touching the nearest point on the train flies back to the pole and then flies towards the train and continues repeating itself. Calculate how much distance will the bird have flown before the train stops?**

Ans. For retardation of train $v^2 - u^2 = 2as$

$$0 = (20)^2 + 2(a)(2)$$

or $$a = -100 \text{ km/hr}^2$$

Time required to stop the train:

$$0 = 20 - 100t$$

or $$t = \frac{1}{5} \text{ hr}$$

For Bird, speed $= \frac{\text{distance}}{\text{time}} = 60 \times \frac{1}{5} = 12$ km

Example 1.9:

Assertion (A): **A body can have acceleration even if its velocity is zero at a given instant.**

Reason (R): **A body is momentarily at rest when it reverses its direction of velocity.**

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

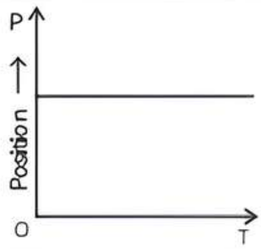
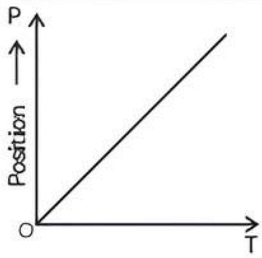
Ans. *(a) Both A and R are true and R is the correct explanation of A.*

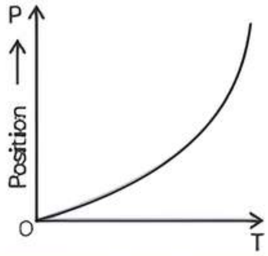
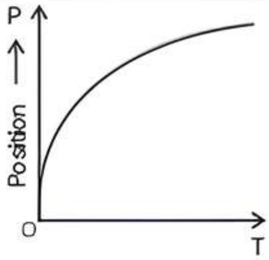
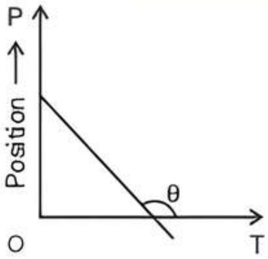
Explanation: When the body reverses the direction of motion it is momentarily at rest, but it still possesses acceleration. Velocity zero, does not mean that acceleration is also zero.

TOPIC 5

GRAPHS

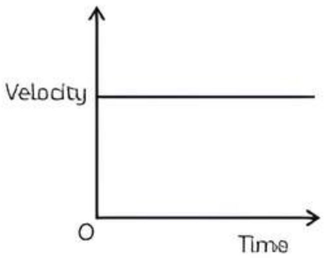
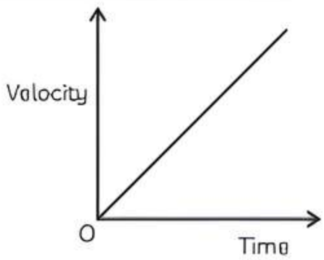
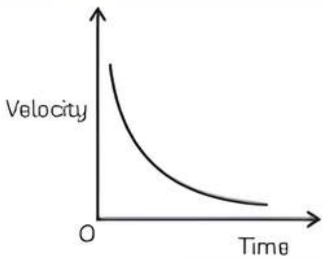
Position-Time Graphs

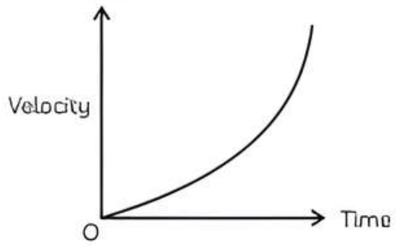
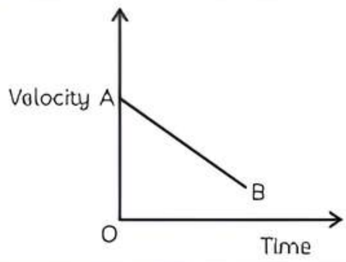
Graph	Condition
	$\theta = 0°$ so $v = 0$ *i.e.*, line parallel to time axis represents that the particle is at rest.
	θ = constant so v = constant, $a = 0$ *i.e.*, line with constant slope represents uniform velocity of the particle.

	θ is increasing so v is increasing, a is positive. *i.e.*, line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.
	θ is decreasing so v is decreasing, a is negative. *i.e.*, line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.
	θ constant but > 90° so v will be constant but negative. *i.e.*, line with negative slope represent that particle returns towards the point of reference. (negative displacement).

Velocity-Time Graph

Slope of the graph represents acceleration

Graph	Condition
	$\theta = 0°$ $\tan\theta = \tan 0° = 0$ Acceleration, $a = 0$ *i.e.*, v = constant or uniform motion
	θ = constant tan θ = constant Acceleration, a = constant *i.e.*, v = uniformly acceleration motion
	θ is decreasing with time tan θ is decreasing with time Acceleration is decreasing with time *i.e.*, acceleration goes on decreasing with time so it is retardation.

	θ is increasing with time tan θ is increasing with time Acceleration is increasing with time *i.e.*, acceleration is increasing with time
	θ > 90° tan θ = negative Acceleration = negative but constant *i.e.*, constant or uniform retardation is acting on the body.

$$\text{Area of } v-t \text{ graph} = \int v\,dt = \frac{\text{Displacement}}{\text{Change in position}}$$

Total area enclosed between speed-time (v-t) graph and time axis represents distance. While vector sum of the total area enclosed between the v-t graph and time axis represents displacement.

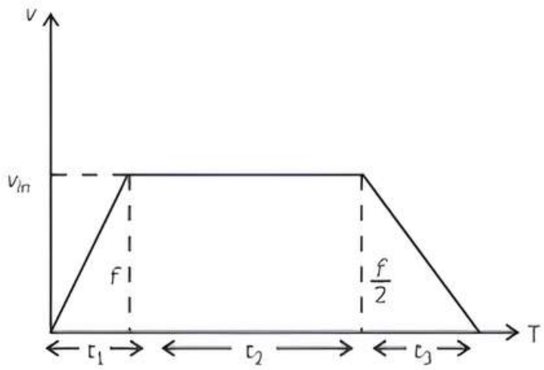
Example 1.10: A car starting from rest, accelerates at the rate of (f) through a distance S then continues at a constant speed for time t and then comes to rest with retardation $\frac{f}{2}$. if the total distance travelled is 15 s then calculate the value of S in terms of f and t.

Ans. Let constant speed be v_{ln} for time t_1,

$$v_{ln} = f t_1 \text{ and } S = \frac{1}{2} f t_1^2$$

For time t_2, $0 = v_{ln}$

$$-v_{ln} - \frac{f}{2} t_2 \text{ or } t_2 = 2t_1$$



$$S_3 = \frac{1}{2}\left(\frac{f}{2}\right)t_2^2 = \left(\frac{f}{4}\right)\left(4t_1^2\right) = f t_1^2 = 2S$$

Therefore,

$$S + v_{ln}t + 2S = 15S$$

$$\Rightarrow \quad v_{ln}t = 12S$$

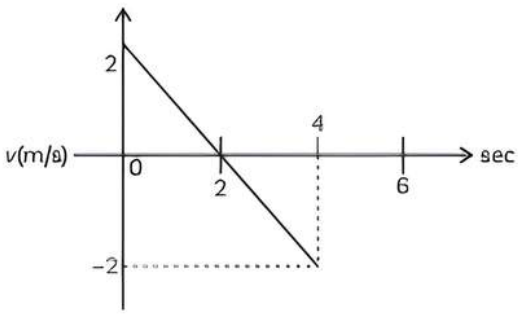
$$\Rightarrow \quad f t_1 t = 12S$$

$$\Rightarrow \quad f\left(\frac{2S}{f}\right)^{1/2} t = 12S$$

$$\Rightarrow \quad 2Sf = \frac{144S^2}{t^2}$$

$$\Rightarrow \quad S = \frac{ft^2}{72}$$

Example 1.11: Velocity – time graph for a particle moving in a straight line is given. Calculate the displacement of the particle and the distance travelled in the first 4 seconds.



Ans. Take the area above the time axis as positive and the area below time axis negative, then

displacement = (2 – 2) m = 0

While displacement takes all areas as positive, the distance covered s = (2 + 2) m = 4 m.

TOPIC 6

THE MOTION UNDER GRAVITY (FREE FALL)

Acceleration produced in a body by the force of gravity is called acceleration due to gravity. It is represented by the symbol g. Value of g = 9.8 m/s^2 = 980 cm/s^2 =32 ft/s^2. In the absence of air, it is found that all bodies (of any size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h << earth's radius) is called motion under gravity. Free fall means acceleration of a body is equal to the acceleration due to gravity. Equations for accelerated motion apply to freely falling bodies. Therefore, equations of motion of freely falling bodies can be obtained by substituting 'g' in place of 'a' in equations for accelerated motion. Thus, for a freely falling body.

(1) $v = u + at$ changes to $v = u + gt$

(2) $s = ut + \frac{1}{2}at^2$ changes to $h = ut + \frac{1}{2}gt^2$

(3) $v^2 - u^2 = 2as$ changes to $v^2 - u^2 = 2gh$

(4) $s(n^{th}) = u + \frac{a}{2}(2n-1)$ changes to

$h(n^{th}) = u + \frac{g}{2}(2n-1)$

Important

➥ *If a body is released or dropped from a certain height, then the initial velocity of the body is taken as zero i.e., u = 0. Hence, Equations of motion of the freely falling body are written as:*

(1) $v = gt$

(2) $h = \frac{1}{2}gt^2$

(3) $v^2 = 2gh$ or $v = \sqrt{2gh}$

(4) $h(n^{th}) = \frac{g}{2}(2n-1)$

g always acts downwards so g is taken as positive i.e., g = + 9.8 m/s^2 and in upwards motion g is taken as negative i.e., g = – 9.8 m/s^2.

Example 1.12: Case Based:

The motion of falling objects is the simplest and most common example of motion with changing velocity. If a coin and a piece of paper are simultaneously dropped side by side, the paper takes much longer to hit the ground. However, if you crumple the paper into a compact ball and drop the items again, it will look like both the coin and the paper hit the floor simultaneously. This is because the amount of force acting on an object is a function of not only its mass but also its area. Freefall is the motion of a body where its weight is the only force acting on an object. The same concept is true for a sky diver and a meteor shower.

(A) A particle is projected vertically upwards from the ground a with velocity of 10 m/s. Find the time taken by it to reach the highest point? [g = 10 m/s^2]

(a) 1 s (b) 2 s
(c) 3 s (d) 4 s

(B) For the above question in Part (A), if the time taken to reach the ground is 5 s then find the height of the tower?

(a) 80 m (b) 75 m
(c) 75.5 m (d) 80.5 m

(C) Suppose you hold a book in one hand and a flat sheet of paper in another hand. You drop them both, and they fall to the ground. Explain why the falling book is a good example of free fall, but the paper is not.

(D) Suppose you throw a ball straight up into the air. Describe the changes in the velocity of the ball. Describe the changes in the acceleration of the ball.

(E) Assertion (A): He tries to decrease the distance travelled by the ball so that it hurts less.

Reason (R): A cricketer moves his hands forward to catch a ball to catch it easily without hurting.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

Ans. (A) *(a) 1 s*

Explanation: Time taken to reach the highest point $= \frac{u}{g}$

$$t = \frac{10}{10} = 1 \text{ s}$$

(B) *(a) 80 m*

Explanation: Time taken for the particle to reach the ground from the highest point is

$$t = 5 - 1 = 4 \text{ s},$$

$$S = \frac{1}{2}gt^2$$

$$= \frac{1}{2}(10)(4)^2$$

$$= 80 \text{ m}$$

Height of tower = 80 – 5 = 75 m

(C) Free fall is the motion of an object when gravity is the only significant force on it. The paper is significantly affected by the air, but the book is not.

(D) Velocity is reduced at a constant rate as the ball travels upward. At its highest point, velocity is zero. As the ball begins to drop, the velocity begins to increase in the negative direction. When it reaches the height from which it was initially released, the ball has the same speed it had upon release. The acceleration is constant throughout the ball's flight.

(E) *(c) A is true but R is false.*

Explanation: A cricket player moves his hands backward to increase the time interval for reducing the momentum of the ball to zero. Thus, the ball does not hit him hard as force is directly proportional to the change of momentum.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. A man walks 30 m towards the North, then 20 m towards the east, and in the last $30\sqrt{2}$ m towards the south-west. The displacement from the origin is:

(a) 10 m towards west

(b) 10 m towards east

(c) $60\sqrt{2}$ m towards north-west

(d) $60\sqrt{2}$ m towards east-north

Ans. *(a) 10 m towards west.*

Explanation: Displacement is given by:

$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$$

$$\vec{d} = 30\hat{j} + 20\hat{i} + 30\sqrt{2}\left(\frac{-\hat{i}-\hat{j}}{\sqrt{2}}\right)$$

$$\vec{d} = 30\hat{j} + 20\hat{i} - 30\hat{i} - 30\hat{j}$$

$$\vec{d} = -10\hat{i}$$

$$\vec{d} = 10 \text{ m towards west.}$$

Caution

➥ *Students must know that if motion is in a straight line without change in direction, then*

Distance = |displacement| = magnitude of displacement

The magnitude of displacement may be equal or less than distance, but never greater than distance.

Distance ≥ displacement

2. A particle moves along the curved path of a quarter circle. The ratio of distance to displacement is:

(a) 11 : 7 **(b) 11 : $7\sqrt{2}$**

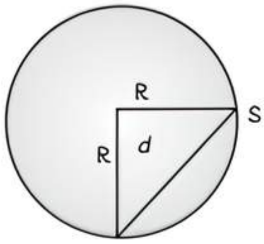
(c) 7 : 11 **(d) 7 : $11\sqrt{2}$**

Ans. *(b) $11 : 7\sqrt{2}$*

Explanation: According to the question

Displacement, $d = R\sqrt{2}$

$$\text{Distances} = \frac{\pi R}{2}$$



So,

$$\frac{s}{d} = \frac{\pi R}{2R\sqrt{2}} = \frac{22}{7 \times 2\sqrt{2}} = \frac{11}{7\sqrt{2}}$$

Related Theory

➥ *Word displacement has 'displace' in it. It can be related to how much a body has been displaced from its initial point to its final point. Displacement and distance can create confusion in a student's mind. With a little wordplay, two values can be differentiated easily.*

3. When the distance travelled by a body is proportional to the time taken. Its speed:

(a) remains unchanged

(b) becomes zero

(c) increases

(d) decreased **[Delhi Gov. SQP 2022]**

Ans. *(a) remains unchanged*

Explanation: Given that,

the distance travelled by a body is proportional to time.

So distance (s) = kt

where k is a constant and t is time.

$$\text{speed } (v) = \frac{ds}{dt} = \frac{d(kt)}{dt} = k \text{ (Constant)}$$

4. In one dimensional motion, instantaneous speed v satisfies $0 \le v \le v_0$ Then:

(a) displacement in time T must always take non-negative values.

(b) displacement x in the time T satisfies $-v_0 T < x < v_0 T$.

(c) acceleration is always a non-negative number.

(d) motion has no turning points.

[NCERT Exemplar]

Ans. *(b) displacement x in the time T satisfies $-v_0 T < x < v_0 T$.*

Explanation: For maximum and minimum displacement, the magnitude and direction velocity.

As maximum velocity, in a positive direction and in opposite direction, is also $-v_0$.

Maximum displacement in one direction = v_0T and in opposite direction is $-v_0T$.

Hence, the range of displacement is given by,

$$-v_0T < x < v_0T.$$

5. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constant. The acceleration is:

(a) $-2abv^2$ (b) $-2bv^3$

(c) $-2av^3$ (d) $-2av^2$

Ans. *(c) $-2av^3$*

Explanation: $t = ax^2 + bx$

Differentiate w.r.t time

$$1 = 2ax\frac{dx}{dt} + b\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{2ax+b}$$

$$v = \frac{1}{2ax+b} \quad \text{...(i)}$$

$$a = \frac{dv}{dt} = \frac{-\left(2a\frac{dx}{dt}\right)}{(2ax+b)^2}$$

$$a = \frac{-2av}{(2ax+b)^2} \quad \text{...(ii)}$$

Substituting $\frac{1}{2ax+b} = v$ in (ii) we will get,

$$a = -2av^3$$

6. An ant is crawling on the rim of a circular path of radius 7 cm. The displacement of the ant in half round is:

(a) 12 (b) 13

(c) 10 (d) 14

[NCERT Exemplar]

Ans. *(d) 14*

Explanation: Given, the radius of circular plank

$$r = 7\text{cm}$$

Distance travelled by ant in half-round

$$= \frac{2\pi r}{2}$$

$$s = \pi r = 3.14 \times 7$$

$$= 21.98 \text{ cm}$$

displacement, $(d) = 2r$

$$= 2 \times 7 = 14 \text{ cm}$$

7. The velocity-time relation of an electron starting from rest is given by $u = kt$ where $k = 2\ m/s^2$. The distance traversed in 3 sec is:

(a) 9 m (b) 16 m

(c) 27 m (d) 36 m

Ans. *(a) 9 m*

Explanation: Velocity is

$$u = kt$$

or $$\frac{ds}{dt} = kt \quad (\because k = 2\ m/s^2)$$

$$\int_0^s ds = 2\int_0^3 t\,dt$$

$$\Rightarrow \quad s = 2\left|\frac{t^2}{2}\right|_0^3$$

$$\Rightarrow \quad s = 9 \text{ m}$$

8. A particle moves along a straight line such that its displacement at any time t is given by $s = t^3 - 6t^2 + 3t + 4$ metres. The velocity when the acceleration is zero is:

(a) 3 m/s (b) −12 m/s

(c) 42 m/s (d) −9 m/s

Ans. *(d) −9 m/s*

Explanation: $s = t^3 - 6t^2 + 3t + 4$

$$v = \frac{ds}{dt} = 3t^2 - 12t + 3 \quad \text{...(i)}$$

$$a = \frac{dv}{dt} = 6t - 12$$

$$\Rightarrow \quad 6t - 12 = 0$$

$$\Rightarrow \quad t = 2 \text{ sec.}$$

Substituting the value in eqn. (i)

$$v = 3 \times (2)^2 - 12 \times (2) + 3$$

$$v = 12 - 24 + 3$$

$$v = -9 \text{ m/s}$$

9. The linear velocity of a body rotating at ω rad/s along a circular path of radius r is given by:

(a) ωr (b) $\frac{\omega}{r}$

(c) $2\omega r$ (d) $\frac{3\omega}{r}$

[Delhi Gov. SQP 2022]

Ans. *(a)* ωr

Explanation: The rate of change of angular position of an object in a rotational/circular motion is called angular velocity.

The SI unit of angular velocity is radian per second.

It is given by the formula:

$$\omega = \frac{d\theta}{dt}$$

where ω = angular velocity in radian per second, $d\theta$ = small angular displacement in radian, and dt = small time in second.

Relation of angular velocity with linear velocity:

$$v = \omega r$$

Where, v = linear velocity of the object, ω = angular velocity of the object and r = radius of the circle in which the body is moving.

10. A body at rest is imparted motion to move in a straight line. It is then obstructed by an opposite force, then:

(a) the body may necessarily change direction.

(b) the body is sure to slow down.

(c) the body will necessarily continue to move in the same direction at the same speed.

(d) none of the above.

Ans. *(b) the body is sure to slow down.*

Explanation: As force is opposite to velocity, so the body will be retarded and slow down.

Related Theory

➥ *The rate of change of velocity with the time of a moving body can either be positive or negative. If the rate of change of velocity with time is positive, it is called acceleration and if negative, it is called retardation. So, when the acceleration is acting in a direction opposite to the velocity, it is called retardation.*

11. A rocket is fired vertically from the ground. It moves upwards with a constant acceleration of 10 m/s^2. After 30 seconds the fuel is finished. After what time from the instant of firing the rocket will it attain the maximum height if g = 10 m/s^2?

(a) 30 s **(b) 45 s**

(c) 60 s **(d) 75 s**

Ans. *(c) 60 s*

Explanation: For upward motion, Velocity after 30 sec,

$$v = u + at$$

$$v = 0 + 10 \times 30$$

$$= 300 \text{ m/s}$$

Now fuel is finished then calculate the time taken to reach maximum height using first equation of motion.

$$v = u + at$$

Where, $v = 0$

$u = 300$ m/s

$a = -10$ m/s^2

$$0 = 300 - 10 \times t$$

$$t = \frac{300}{10} = 30 \text{ s}$$

So total time, T = 30 + 30 = 60 s

Related Theory

➥ *Actual value of gravity is taken to be 9.8 m/s^2 but for the sake of calculation, it is taken as an even figure, 10 m/s^2. In various space organization, accurate value of gravity is taken for calculation as they cannot afford even a minor error. Check out the difference between precise value and accurate value in previous chapter.*

12. A body dropped from the top of the tower of height H meters. It takes 't' time to reach the ground. Where is the body, $\frac{t}{2}$ times after the release?

(a) At $\frac{H}{2}$ meters from the ground

(b) At $\frac{H}{4}$ meters from the ground

(c) At $\frac{3H}{4}$ meters from the ground

(d) At $\frac{H}{6}$ meters from the ground

Ans. *(c) At $\frac{3H}{4}$ meters from the ground*

Explanation: We know for free fall

$$H = \frac{1}{2}gt^2$$

$$h = \frac{1}{2}g\left(\frac{t}{2}\right)^2$$

$$= \frac{H}{4}$$

from releasing point.

Hence, remaining distance from ground,

$$s = H - \frac{H}{4}$$

$$= \frac{3H}{4}$$

13. A car moving with a speed of 50 km/h can be stopped by brakes after at least 6 m. If the same car is moving to speed of 100 Km/h the minimum stopping distance:

(a) 12 m **(b) 18 m**

(c) 24 m **(d) 6 m** [Diksha]

Ans. *(c) 24 m*

Explanation: Given,

$$u = 50 \text{ km/hr}$$

$$= 50 \times \frac{5}{18} = \frac{250}{18} \text{ m/s}$$

$$v = 0$$

$$s = 6 \text{ m (at least)}$$

By third equation by motion,

$$v^2 = u^2 + 2as$$

$$0 = \left(\frac{250}{18} \times \frac{250}{18}\right) + 2(-a)(s)$$

$$0 = 192.90 - 12a$$

$$a = \frac{192.90}{12}$$

$$= 16.07$$

Now applying third equation of motion,

$$v^2 = u^2 + 2as$$

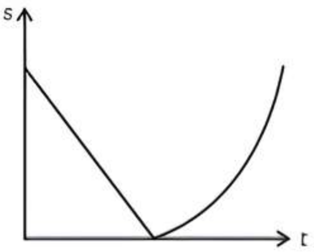
$$\Rightarrow \quad 0 = \left(100 \times \frac{5}{18}\right)^2 + 2(-16.07)(s)$$

$$\Rightarrow \quad 0 = 771.60 - 32.14s$$

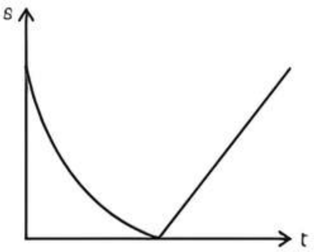
$$\Rightarrow \quad s = \frac{771.60}{32.14} = 24.00$$

14. A ball is thrown vertically upwards. Which of the following plot represents the speed-time graph of the ball during its flight, if the air resistance is not ignored?

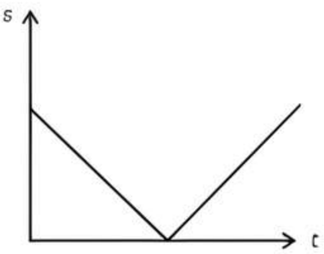
(a)



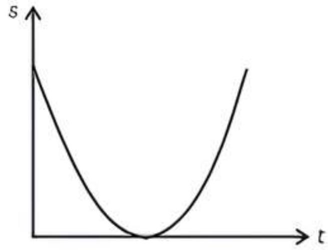
(b)



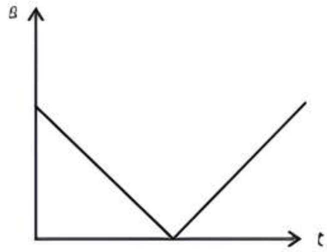
(c)



(d)

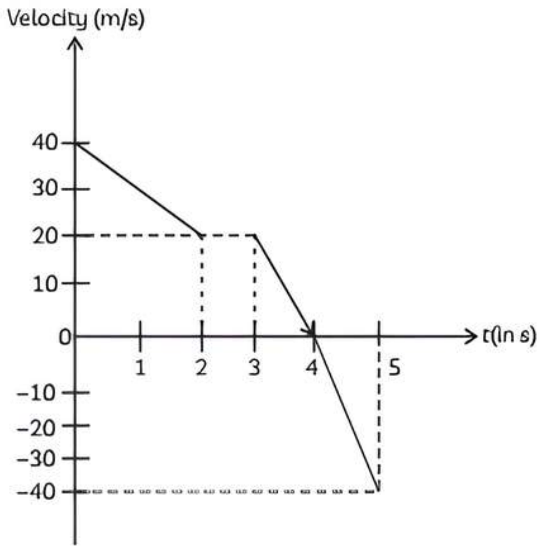


Ans. (c)



Explanation: In the first half of motion, the acceleration due to gravity g is uniform or constant & velocity gradually decreases, so slope m will be negative but for the next half, acceleration is positive. So, the slope m will be positive. Thus, option (c) is correct.

15. From the following velocity-time graph of a body, the distance travelled by the body and its displacement during 5 seconds in meters will be:



(a) 75,75 **(b)** 110,70
(c) 110,110 **(d)** 110,40

Ans. *(b) 110,70*

Explanation: Distance $= \frac{1}{2}(40+20)\times 2 + (20\times 1)$

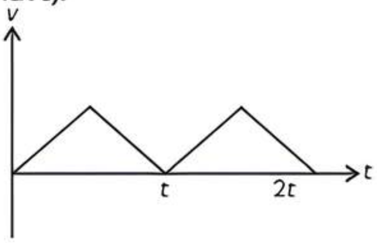
$$+\frac{1}{2}(20\times 1) + \frac{1}{2}(20\times 1) + \frac{1}{2}(40\times 1) = 110 \text{ m}$$

$$\text{Displacement} = \frac{1}{2}(40+20)\times 2 + (20\times 1)$$

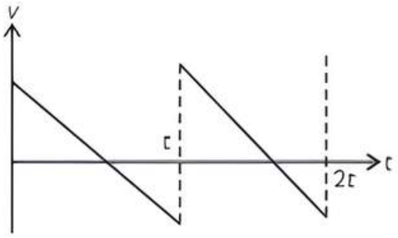
$$+\frac{1}{2}(20\times 1) - \frac{1}{2}(40\times 1) = 70 \text{ m}$$

16. A ball is dropped from a certain height on the surface of the glass. It collides elastically and comes back to its initial position. If this process is repeated, then the velocity-time graph is (considering downward direction as positive):

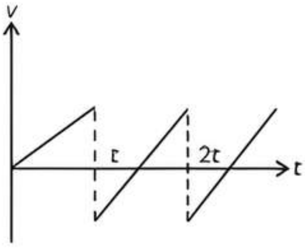
(a)



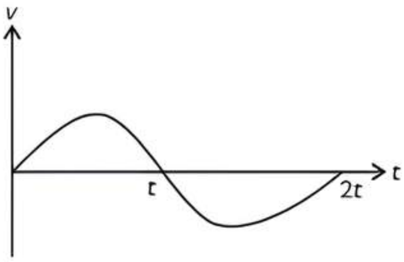
(b)



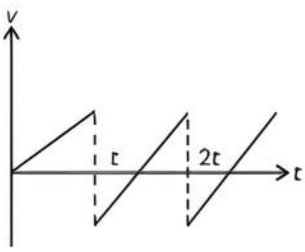
(c)



(d)



Ans. (c)



Explanation: Acceleration is always downward *i.e., positive*, so the slope of the *v-t curve* will always be positive.

Assertion-Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these question from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

17. Assertion (A): In real life, in a number of situations, the object is treated as a point object.

Reason(R): An object is treated as a point object, as far as its size is much smaller than the distance, it moves in a reasonable duration of time.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: The approximation of an object as a point object is valid only when the size of the object is much smaller than the distance it moves in a reasonable duration of time.

18. Assertion (A): The actual distance covered by an object in a given time interval can be equal or greater than the magnitude of the displacement.

Reason(R): The distance covered is a scalar quantity, while the displacement is a vector quantity.

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: The actual distance covered by an object in the given time interval is equal to the magnitude of the displacement when the object moves along a straight path in one fixed direction.

19. Assertion (A): The speed of a body can be negative.

Reason(R): If the body is moving in the opposite direction of positive motion, then its speed is negative.

Ans. *(d) A is false and R is also false.*

Explanation: Speed can never be negative because it's a scalar quantity. So if a body is moving in a negative direction, then also the speed will be positive.

20. Assertion (A): An object may have varying speeds without having varying velocity.

Reason(R): If the velocity is zero at an instant, the acceleration is zero at that instant.

Ans. *(d) A is false and R is also false.*

Explanation: If the speed varies, then velocity will definitely vary when a particle is thrown upwards at the highest point $a \neq 0$ but $v = 0$

21. Assertion (A): For an object having uniformly accelerated motion, the position-time graph is parabolic in nature.

Reason(R): In a uniformly accelerated motion, the acceleration is constant

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: In a uniformly accelerated motion, the distance (x) covered in a time t is given by

$$s = ut + \frac{1}{2}at^2.$$

It represents that its parabola.

22. Assertion (A): For an object in uniform motion, the velocity-time graph is a straight line parallel to the time axis.

Reason(R): In a uniform motion, velocity remains constant

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: In a uniform motion velocity does not change with time and according to the velocity-time graph is a straight line parallel to the time axis.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

23. *For over speeding, police use speed traps to avoid accidents yet some people find a way to avoid speed traps. Many do so by slowing down before approaching the camera. Over speeding is a major issue on the long and wide highways but the authority still finds a way to fine the over speeders. The toll booth on highways knows the total distance of the highway and when a car approaches the booth, they check the time when the car got on the highway and when they are getting off the highway. By this method they deduce the speed with which the car travelled and fine the over speeders.*

(A) A bike rider is travelling from Gurgaon to Noida. In between his entire travelling he crossed a circular path whose radius is r and reminds his old days that during his childhood how he enjoyed all this and noticed that he took around 40 s in covering one revolution. After reaching home he narrated the entire thing to her daughter to which she asked if he revolved around that circular path for around 2 min 20 s then how much distance he has covered and what is his displacement?

(B) The distance travelled by a body is found to be directly proportional to the square of time. Is the body moving with uniform velocity or with uniform acceleration if the distance travelled is directly proportional to time?

(C) The displacement of a body is given to be proportional to the cube of time elapsed. What is the nature of acceleration of the body?

Ans. (A) If t = 2 min 20 s

i.e., 1 min = 60 s

2 min = 60 × 2 = 120 s

So, total time, t = 120 s + 20 s = 140 s

Number of rotations, $\frac{140}{40} = 3 + \frac{1}{2}$

Displacement in $\frac{1}{2}$ rotation = 2r

Distance in $3 + \frac{1}{2}$ rotation

= 3 × 2πr + πr

= 7πr

(B) The body is moving with uniform acceleration if the distance travelled is directly proportional to the square of time, and if it is proportional only to time, it is moving with uniform velocity.

(C) As $s \propto t^3$ or $s = kt^3$

Velocity, $v = \frac{ds}{dt} = 3kt^2$

Acceleration, $a = \frac{dv}{dt} = 6kt$

i.e., acceleration ∝ time

Clearly, the acceleration increases uniformly with time.

24. *An object released near the surface of the Earth is accelerated downward under the influence of the force of gravity. The magnitude of acceleration due to gravity is represented by g. If air resistance is neglected, the object is said to be in free fall.*

If the height through which the object falls is small compared to the earth's radius, g can be taken to be constant, equal to 9.8 m/s^2.

Free fall is thus a case of motion with uniform acceleration. We assume that the motion is in y-direction, more correctly in -y-direction because we choose upward direction as positive.

Since the acceleration due to gravity is always downward, it is in the negative direction and we have $a = -g = -9.8\ ms^{-2}$. The object is released from rest at $y = 0$. Therefore, $v_0 = 0$ and the equations of motion become

$$v = 0 - gt = -9.8\ t\ ms^{-1}$$

$$y = 0 - \frac{1}{2}gt^2 = -4.9\ t^2\ m$$

$$v^2 = 0 - 2\ gy = -19.6\ y\ m^2s^{-2}$$

These equations give the velocity and the distance travelled as a function of time and also the variation of velocity with distance.

[Delhi Gov. SQP 2022]

(A) Suppose you hold a book in one hand and a flat sheet of paper in another hand. You drop them both, and they fall to the ground. The falling book is a good example of free fall, but the paper is not because:

(a) The book is significantly affected by the air.

(b) The paper is relatively more affected by gravity.

(c) Free fall is the motion of an object when gravity is the only significant force on it. The paper is significantly affected by the air, but the book is not.

(d) None of the above.

(B) Suppose you throw a ball straight up into the air. The correct option is:

(a) Velocity is reduced at a constant rate as the ball travels upward.

(b) At its highest point, velocity is zero.

(c) As the ball begins to drop, the velocity begins to increase in the negative direction.

(d) All of the above

(C) A stone that starts at rest is in free fall for 8.0 s. The stone's velocity after 8.0 s will be:

(a) 78.4 m/s downward

(b) 108 m/s downward

(c) 118 m/s downward

(d) 97 m/s downward

(D) A stone that starts at rest is in free fall for 8.0 s. The stone's displacement during this time will be:

(a) 510 m downward

(b) 800 m downward

(c) 100 m downward

(d) 310 m downward

(E) Assertion (A): At any instant, the acceleration of a body can change its direction without any change in direction of velocity.

Reason (R): At any instant, the direction of acceleration is the same as that of the direction of change in velocity vector at that instant.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

Ans. (A) *(c) Free fall is the motion of an object when gravity is the only significant force on it. The paper is significantly affected by the air, but the book is not.*

Explanation: The motion of an item when gravity is the only substantial force acting on it is referred to as free fall. The air has a considerable impact on the paper, but not on the book.

(B) *(d) All of the above*

Explanation: As the ball moves higher, its velocity decreases at a steady pace. Velocity is 0 at its maximum point. As the ball begins to fall, the velocity increases in the opposite direction.

(C) *(a) 78.4 m/s downward*

Explanation: Given: Initial velocity,

u = 0 m/s (since, the stone starts at rest)

Time, t = 8 seconds

Acceleration due to gravity,

a = 9.8 m/s^2. Mathematically, the first equation of motion is given by the formula:

$$v = u + at$$

$$v = 0 + 9.8 \times 8$$

Velocity, v = 78.4 m/s

(D) *(d) 310 m downward*

Explanation: Given:

Initial velocity, = 0 m/s

(since, the stone starts at rest)

Time, t = 8 seconds

Acceleration due to gravity, a = 9.8 meter per seconds square of motion:

$$S = ut + \frac{1}{2}at^2$$

$$S = 0(8) + \frac{1}{2}(9.8)(8)^2$$

$$S = 0 + \frac{1}{2}(9.8)(64)$$

$$S = 4.9 \times 64$$

Displacement, S = 313.6 meters

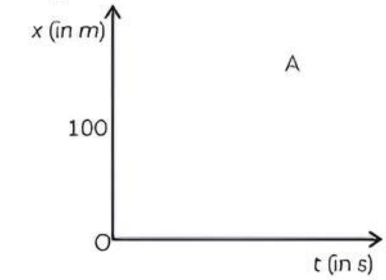
(E) *(b) Both A and R are true and R is not the correct explanation of A.*

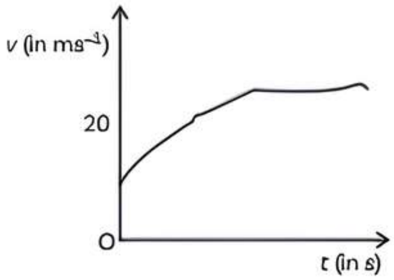
Explanation: Acceleration depends upon the applied force.

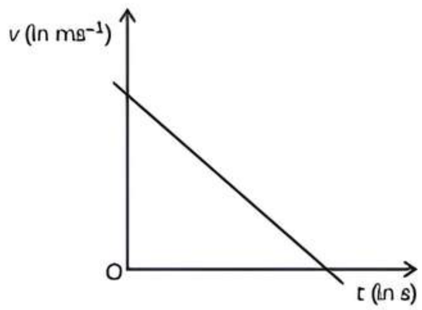
As, we know that, F = ma

So, $$a = \frac{F}{m}$$

25. **A driver is driving a car on a straight road. The car is moving with a velocity of 20 ms^{-1} at t = 0 along the x – axis. At t = 0, the car passes a milestone at x = 100 m. The acceleration of the car varies with time as a = (6 – 0.2t) ms^{-2}. The x – t, v – t, and a – t graphs of the motion of the car are shown in figure:**







(A) The velocity of the car after 5 seconds is:

(a) 35.0 ms^{-1} **(b) 35.5 ms^{-1}**

(c) 37.5 ms^{-1} **(d) 47. 5 ms^{-1}**

(B) The distance travelled by car after 5 seconds is:

(a) 295.0 m **(b) 295.5 m**

(c) 270.8 m **(d) 276.5 m**

(C) The time after which the car attains maximum velocity is:

(a) 10 s **(b) 20 s**

(c) 25 s **(d) 30 s**

(D) The maximum velocity attained by the car is:

(a) 120 ms^{-1} **(b) 140 ms^{-1}**

(c) 150 ms^{-1} **(d) 110 ms^{-1}**

(E) A displacement time graph of two particles A and B are straight lines making angles of 30° and 60° respectively with the time axis. If the velocities of these particles are v_a and v_b respectively, then the value of $\frac{v_a}{v_b}$ is:

(a) $\sqrt{3}$ **(b) $\frac{1}{\sqrt{3}}$**

(c) $\frac{1}{3}$ **(d) 3**

Ans. (A) *(d) 47.5 ms^{-1}*

Explanation:

$$v = 20 + \int_0^t a\,dt$$

$$= 20 + \int_0^5 (6 - 0.2t)\,dt$$

$$= 20 + \left[6t - 0.1t^2\right]_0^5$$

$$= 20 + 30 - 2.5 = 47.5\ \text{ms}^{-1}$$

(B) *(c) 270.8 m*

Explanation:

$$s = x + \int_0^t v\,dt$$

$$= 100 + \int_0^5 (20 + 6t - 0.1t^2)\,dt$$

$$= 100 + \left[20t + 3t^2 - \frac{0.1t^3}{3}\right]_0^5$$

$$= 100 + 100 + 75 - 4.17$$

$$= 270.8\ \text{m}$$

(C) (d) 30 s

Explanation:

Since, $a = (6 - 0.2t)$

or $\frac{dv}{dt} = 6 - 0.2t$

$6 - 0.2t = 0$

As v is constant.

So, $\frac{dv}{dt} = 0$

Then, $t = 30$ s

(D) (d) $110\ ms^{-1}$

Explanation:

$v_{max} = 20 + 6t - 0.1 \times t^2$
$= 20 + 6 \times 30 - 0.1 + 900$
$= 110\ ms^{-1}$

(E) (c) $\frac{1}{3}$

Explanation: Velocity of a particle = slope of the displacement-time graph = tan θ

$v_a = \tan\theta = \tan 30°$

And $v_b = \tan\theta = \tan 60°$

$\frac{v_a}{v_b} = \frac{\tan 30°}{\tan 60°} = \frac{1}{\sqrt{3} \times \sqrt{3}} = \frac{1}{3}$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

26. **A 400 m long railway train is going from New Delhi Railway Station to Isanpur. Can we consider the railway station as a point object?**

Ans. Yes, because the length of the railway station is smaller as compared to the distance between New Delhi and Isanpur.

27. **Shipra went from her home to school, 2.5 km away. On finding her school closed, she returned to her home immediately. What is her net displacement? What is the total distance covered by her?**

Ans. Total distance is 2.5 km + 2.5 km = 5.0 km, but net displacement is zero, because her initial and final point remains same.

28. **A person standing on a tower throws a stone vertically upward with same velocity *u* and drops another one downward with the same initial velocity. Which stone will strike the earth with a larger velocity?**

Ans. Both stones will strike with the same velocity, but the stone, thrown vertically upwards will take $\frac{t}{2}$ sec more than the other one.

29. **Can the earth be regarded as a point object when it is describing its yearly journey around the sun? [Diksha]**

Ans. Yes because the size of the Earth is much smaller than the distance from the Sun.

30. **Is it possible for a body to be accelerated without speeding up or slowing down? If so, give an example.**

Ans. Yes. An object in uniform circular motion is accelerating but its speed is neither decreases nor increases.

31. **Even when rain is falling vertically downwards, the front screen of a moving car gets wet while the back screen remains dry. Why?**

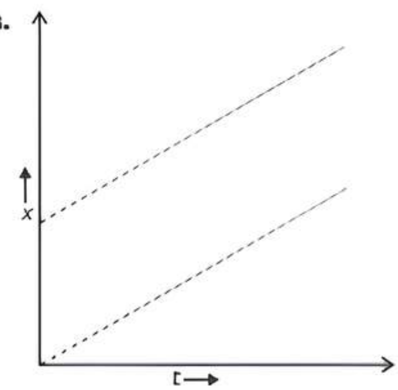
Ans. This is because the rain strikes the car in the direction of the relative velocity of rain with respect to the car.

32. **Suggest a situation in which an object is accelerated and have constant speed.**
[Delhi Gov. QB 2022]

Ans. When a body moves on a circular course, its speed remains constant but its direction changes, causing it to accelerate.

33. **Draw Position-time graph of two objects, A and B moving along a straight line, when their relative velocity is zero.**
[Delhi Gov. QB 2022]

Ans.



Position-Time Graph

34. Can the speed of a body change, if its velocity is constant? [NCERT Exemplar]

Ans. No, because the speed of a particle is equal to the magnitude of its velocity.

35. The average velocity of a particle is equal to its instantaneous velocity. What is the nature of its displacement-time graph?

Ans. When the average velocity is equal to the instantaneous velocity, the particle has a uniform motion. So, its displacement-time graph is a straight line inclined to the time axis.

36. What is the nature of the displacement time curve of a body moving with constant velocity?

Ans. It is a parabola. A straight line inclined to the positive direction of the x-axis in the distance-time graph will give velocity(constant).

Related Theory

➥ *A parabolic curve in the displacement time graph represents acceleration (constant) if the direction of the curve does not change.*

37. Can the direction of motion of a body change, if its velocity is changing at a uniform rate?

Ans. Yes. This can happen in a retarded motion. A body was thrown up moves under constant retardation and from the highest point, it begins to fall downwards.

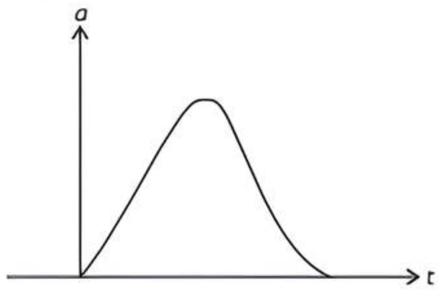
38. Which of the two- linear velocity or the linear acceleration gives the direction of motion of body? [Delhi Gov. QB 2022]

Ans. The direction of motion is determined by velocity.

The acceleration of the body does not define its direction of motion, although the velocity of the body does. The sign of acceleration shows whether the velocity is growing or decreasing, whereas the sign of velocity denotes the direction of motion.

39. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time interval. Show the variation of its acceleration with time.

Ans. When the ball is hit with the bat, its acceleration will increase with time. After the contact between the ball and the bat is over, its acceleration will decrease with time. The variation of acceleration of the ball with the time as shown.



SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

40. A juggler maintains four balls in motion, making each in turn rise to a height of 20 m from his hand. With what velocity does he project them, and where will the other three balls be at the instant when the fourth one is just leaving the hand?

Ans. For the upward motion of a ball:

$v = 0$, $a = -10\ ms^{-2}$, $s = 20$ m, $u = ?$, $t = ?$

As $v^2 - u^2 = 2as$;

$\therefore$ $0 - u^2 = -2 \times 10 \times 20$

$u = 20\ ms^{-1}$

Also $v = u + at$

$0 = 20 - 10t$

$t = 2$ s

So, the ball returns to the hand of the juggler after 4s.

To maintain proper distance, the balls must be thrown up at an interval of $\frac{4}{4} = 1$s

When the fourth ball is in hand, the third ball has travelled for 1 s, second for 2 s, and first for 3 s.

(1) For third ball,

$$s = ut + \frac{1}{2}at^2$$
$$= 20 \times 1 - \frac{1}{2} \times 10 \times (1)^2$$
$$= 15 \text{ m}$$

(2) For the Second ball,

$$s = ut + \frac{1}{2}at^2$$
$$= 20 \times 2 - \frac{1}{2} \times 10 \times (2)^2$$
$$= 20 \text{ m}$$

(3) For first ball,

$$s = ut + \frac{1}{2}at^2$$
$$= 20 \times 3 - \frac{1}{2} \times 10 \times (3)^2$$
$$= 15 \text{ m}$$

41. Two trains 120 m and 80 m in length are running in opposite directions with velocities 42 kmh^{-1} and 30 kmh^{-1}. In what time they will completely cross each other?

Ans. Relative velocity of one train with respect to the other

$$= 42 - (-30) = 72 \text{ kmh}^{-1} = 20 \text{ ms}^{-1}$$

Total distance to be travelled by each train to cross other train $= 120 + 80 = 200$ m

Time is taken by each train to cross other train.

$$= \frac{200}{20} = 10 \text{ s.}$$

42. The speed of a motor launch with respect to still water = 7 ms^{-1} and the speed of the stream is u = 3 ms^{-1} When the launch began travelling upstream, afloat 'was dropped from it. The launch travelled 4.2 km upstream, turned about, and caught up with the float. How long is it before the launch reaches the float?

Ans. For the upstream motion of launch.

Relative Velocity = 7–3 = 4 ms^{-1}

Distance moved = 4.2 km = 4200 m

Time taken. $t_1 = \frac{4200}{4} = 1050$ s

For the downstream motion of launch

Distance moved downstream by float in the

1050 s = 3 × 1050

= 3150 s

Distance between float and launch turned about

= 4200 + 3150

= 7350 m

The distance covered by launch to its velocity (7 ms^{-1}) because stream velocity is shared by both.

Total time taken.

$$t_2 = \frac{7350}{7}$$

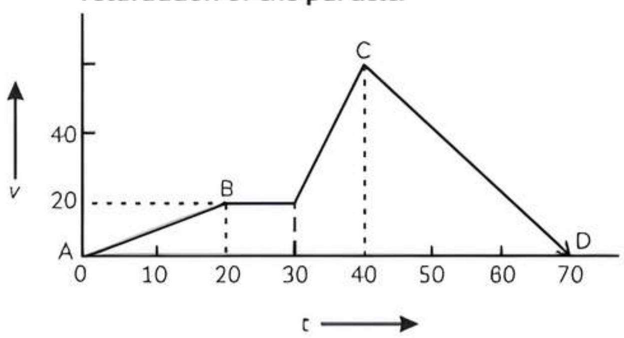
= 1050 s

Total time taken. $t = t_1 + t_2$

= 1050 + 1050

= 2100 s

43. The velocity versus time curve of a moving point is shown in the figure. Find the retardation of the particle.

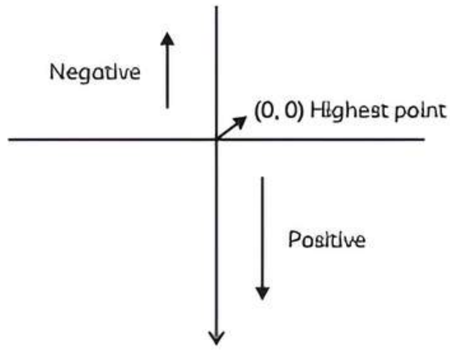


Ans. The slope of the velocity-time graph represents the acceleration or retardation of the particle during motion. If the slope is positive, it represents acceleration and if the slope is negative, it represents retardation. The section CD of the graph represents retardation and the magnitude of retardation is.

$$\vec{a} = \frac{\text{change in velocity}}{\text{time}}$$

$$= \frac{60}{70-40} = 2 \text{ ms}^{-2}$$

44. A ball is thrown upwards with an initial velocity of 10 ms^{-1}. Considering the highest point as the origin and vertically downward direction as the positive direction, find the signs of position, velocity, and acceleration of the object under motion during its upward and downward journey.



Ans. As the particle always remains below (0, 0) and the downward direction is positive, so position x will remain positive during both upwards and downwards motion. During upward motion, velocity is negative and during downward motion velocity is positive. Acceleration is positive during both upward and downward motion. It is because the acceleration due to gravity (g) is downward and our downward direction is positive.

45. A child throws a ball up with an initial velocity of 20 ms^{-1}. Find out the maximum height that the ball can achieve and how long will it take to come back to the child's hands?

Ans. Initial velocity, u = 20 m/s, final velocity, v = 0. The final velocity is zero as the ball must be temporarily at rest at the maximum height. We take the point of projection as the origin and the upward direction as the positive direction.

Maximum height = s = ?, Time = ?

$$v^2 - u^2 = 2as$$

which gives, $0^2 - 20^2 = 2(-10)$ s

or $s = 20$ m

Here we have taken $a = -g = -10$ ms^2 because the upward direction is positive. So the maximum height attained is 20 m.

For time, $v = u + at$

$0 = 20 - 10t$

or $t = 2$ s.

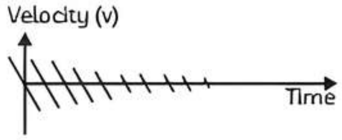
It means the ball went up to the top in 2 s. For motion under gravity, the time of ascent and the time of descent is exactly equal. (Provided air resistance is negligible.)

So, the total time of travel T = (2 + 2)s = 4 s.

46. What are positive and negative acceleration in straight line motion? [Delhi Gov. QB 2022]

Ans. If an object's speed rises with time, its acceleration is positive. (Acceleration is in the direction of motion) If an object's speed decreases with time, its acceleration is negative (Acceleration is opposite to the direction of motion).

47. Suggest a suitable physical situation for the following graph.



[Delhi Gov. QB 2022]

Ans. The graph indicates that the velocity drops from a positive to a negative number and then abruptly declines. The process depicted in the graph is the motion of a ball falling from a height onto the ground and bouncing again.

48. An object is in uniform motion along a straight line, what will be position time graph for the motion of object, if

(A) x_0 = positive, v = negative $|\vec{v}|$ is constant.

(B) both x_0 and v are negative $|v|$ is constant.

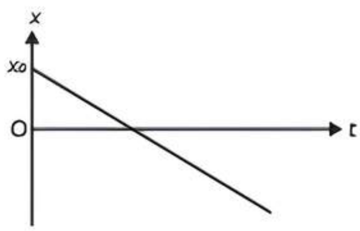
(C) x_0 = negative, v = positive $|\vec{v}|$ is constant.

(D) both x_0 and v = positive $|\vec{v}|$ is constant.

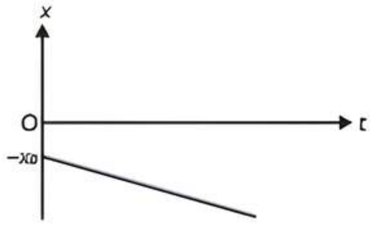
where x_0 is position at $t = 0$

[Delhi Gov. QB 2022]

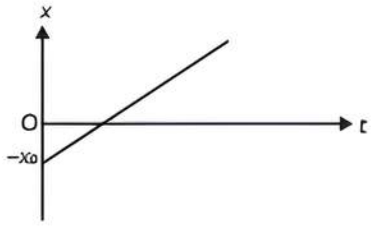
Ans. (A) x_0 = positive, v = negative $|\vec{v}|$ is constant.



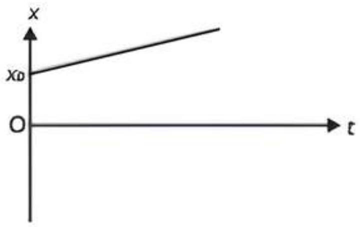
(B) Both x_0 and v are negative $|v|$ is constant.



(C) x_0 = negative, v = positive $|\vec{v}|$ is constant.



(D) Both x_0 and v = positive $|\vec{v}|$ is constant.



49. The greatest height to which a man can throw a stone is h. What will be the greatest distance upto which he can throw the stone? [Delhi Gov. QB 2022]

Ans. The maximum vertical height that the stone achieves = h

Thus, the velocity with which the objects thrown upwards $v = \sqrt{2gh}$

We know that, the range of projectile,

$$R = \frac{v^2 \sin 2\theta}{g} = \frac{v^2}{g}$$

Maximum distance can be covered when $\theta = 45°$ and $\sin 2\theta = 1$

Therefore, $R = \frac{2gh}{h} = 2h$

50. A gunman always keep his gun slightly tilted above the line of sight while shooting. Why? [Delhi Gov. QB 2022]

Ans. When a bullet is shot from a gun with the barrel aimed at the target, it begins to descend downward owing to gravity's acceleration. As a result, the bullet misses the target. To avoid this, the gun's barrel is aligned slightly above the target, so that the bullet, after travelling in a parabolic route, strikes the far target.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

51. A ball is dropped from the top of a high building at $t = 0$. At a later time, $t = t_0$, a second ball is thrown downward with initial speed v_0. Obtain an expression for the time t at which the two balls meet.

Ans. Let the distance covered by the first ball = Y_1

Similarly,

the distance covered by the second ball = Y_2

We can use the condition that at a particular time (as given) distances covered by both the balls are the same. But the caution is that the first ball has covered Y_1 into time and the second ball has covered Y_2 in $(t - t_0)$ time.

Let us assume that the origin is at the top of the building with a downward direction positive. If the two balls meet at time t, then

For Y_1, $t = t_0$, $v_0 = 0$, $a = g$.

$$Y_1 = \frac{1}{2}gt_0^2$$

For Y_2, $t = t - t_0$, initial speed = v_0, $a = g$

$$Y_2 = v_0(t - t_0) + \frac{1}{2}g(t - t_0)^2$$

From the given condition,

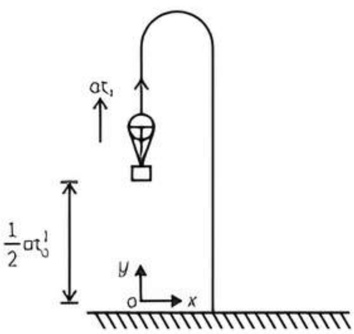
$$Y_1 = Y_2$$

$$\frac{1}{2}gt^2 = v_0(t - t_0) + \frac{1}{2}g(t - t_0)^2$$

After simplifying and solving, we get

$$t = \left[\frac{v_0 - \frac{gt_0}{2}}{v_0 - gt_0}\right]t_0$$

52. A balloon starts rising upward with a constant acceleration and after time t_0 second, a packet is dropped from it which reaches the ground after t second as shown in figure. Determine the value of t.



Ans. Analysis of situation: $t = 0$ is the time when the balloon started rising up. At, $t = t_0$ when the packet is dropped. the balloon is moving up with velocity,

$$v = 0 + at_0 = at_0$$

Hence, initial velocity of the packet will be $v_0 = at_0$ (upward). As the balloon has started rising upwards with constant acceleration, a, so after t_0 seconds its height from the ground is

$$y_0 = \frac{1}{2}gt^2$$

$$s = ut + \frac{1}{2}at^2$$

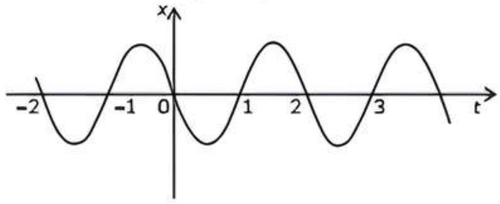
$$-\frac{1}{2}at^2 = at_0t - \frac{1}{2}gt^2$$

$$gt^2 - 2at_0t - at_0^2 = 0$$

Solving the quadratic equation, we get

$$t = \frac{at_0}{g}\left[1 + \sqrt{1 + \frac{g}{a}}\right]$$

53. In the given figure gives the $x - t$ plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity, and acceleration variables of the particle at $t = 0.3$ s, 1.2 s, - 1.2 s.



Ans. The acceleration of a particle executing S.H.M. is given by,

$$a = -\omega^2x$$

Where, ω (angular frequency) is a constant.

At time, $t = 0.3$ s

As is obvious from the graph, $x < 0$

As slope of x-t graph is negative, so $v < 0$

As $a = -\omega^2x$, so $a > 0$.

At time, $t = 1.2$ s

As is obvious from the graph, $x > 0$

As slope of $x - t$ graph is positive, so $v > 0$

As $a = -\omega^2x$, so $a < 0$.

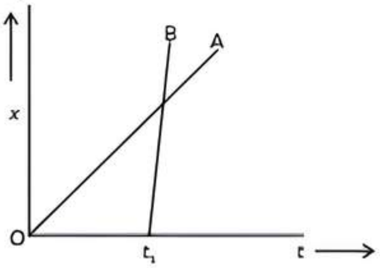
At time, $t = -1.2$ s

As is obvious from the graph, $x < 0$

As slope of $x - t$ graph is positive, so $v > 0$

As $a = -\omega^2x$, so $a > 0$.

54. **Two girls A and B are coming back to their homes, P and Q, respectively from a school O. The position-time graph (*i.e.*, *x*-*t* graph) of their motion is shown in the figure. Find whether A/B lives closer to the school than B/A and A/B starts earlier from the school than B/A. Also, find who is walking faster A/B or B/A.**



Ans. The distance of P from O is less than the distance of Q from O. Hence girl A lives closer to the school than girl B.

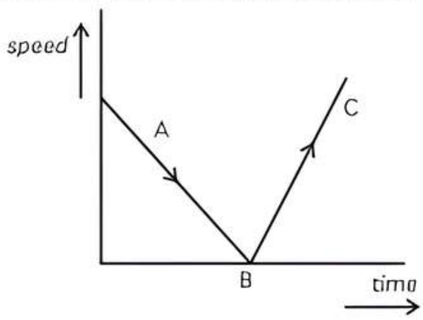
Since, girl B starts her journey after time t_1 then girl A starts her journey. Therefore, girl A starts earlier from school than girl B.

The speed of a body is equal to the slope of the *x*-*t* graph then the motion of girl B is steeper than the motion of girl A so girl B walks faster than A.

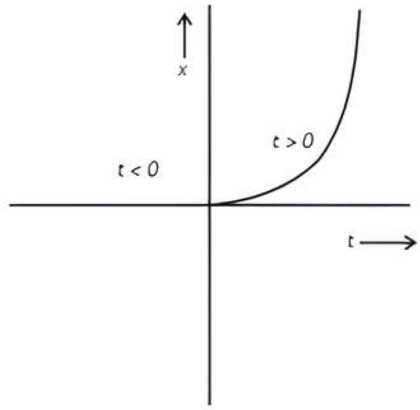
55. **A ball is thrown vertically upwards after some time it returns back to earth. Draw its speed-time graph locating the motion of the ball.**

Ans. When the ball is thrown vertically upward, it has some initial speed as it goes up its speed decreases and becomes zero at the highest point. From that point, it starts the downward journey. The speed goes on increasing and becomes equal to the initial speed when the ball reaches the ground.

The motion of the ball is represented as:



56. **The figure shows the *x*-*t* plot of the one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.**



Ans. No, it is wrong to say that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$, because a position-time $(x - t)$ graph does not represent the trajectory of a moving particle.

This graph can represent the motion of a freely falling particle dropped from a tower when we take its initial position as $x = 0$, at $t = 0$.

57. **The position of an object is given by $x = 2t^2 + 3t$. Find out that its motion is uniform or non uniform.**

[Delhi Gov. SQP 2022]

Ans. Given that:

$$x = 2t^2 + 3t$$

Differentiating w.r.t. time,

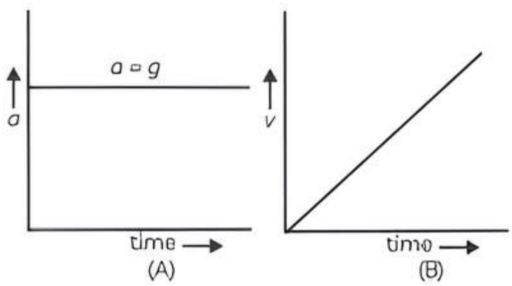
$$v = \frac{dx}{dt} = 2 \times 2t + 3$$

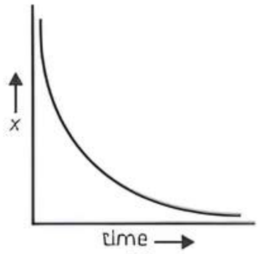
$$v = 4t + 3$$

From this, we can say that velocity of object varies linearly with time. Thus, this is an example of non-uniform motion.

58. **Draw (A) acceleration – time (B) velocity – time (C) Position – time graphs representing motion of an object under free fall. Neglect air resistance.** **[Delhi Gov. QB 2022]**

Ans. The object falls with uniform acceleration equal to '*g*'





(C)

59. For an object projected upward with a velocity v_0, which comes back to the same point after some time, draw:

(A) Acceleration-time graph

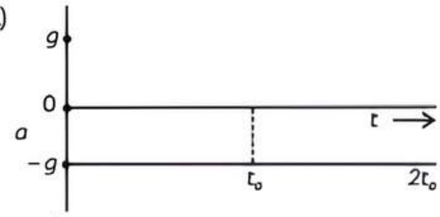
(B) Position-time graph

(C) Velocity-time graph

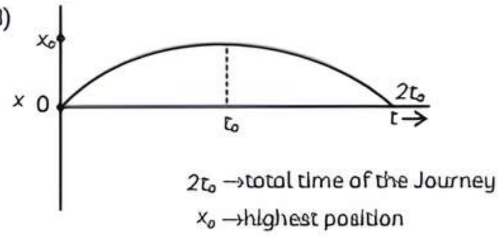
[Delhi Gov. QB 2022]

Ans. Acceleration-time graph.

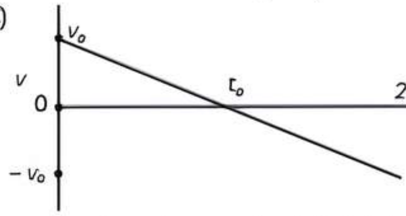
(A)



(B)



(C)



60. A car moving along a straight highway with speed of 126 km/h^{-1}, is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop? [Delhi Gov. QB 2022]

Ans. Given: $u = 126 \text{ km/h} = 126 \times \frac{5}{18} \text{m/s} = 35 \text{ m/s}$

$$v = 0$$

$$s = 200 \text{ m}$$

Newton's Equation of motion

$$v^2 - u^2 = 2as$$

$$0^2 - 35^2 = 2a(200)$$

$$a = -3.0625 \text{ m/s}^2$$

Also

$$v = u + at$$

$$0 = 35 - 3.06t$$

$$t = 11.4 \text{ s}$$

61. The data regarding the motion of two different objects P and Q is given in the following table. Examine them carefully and state whether the motion of the objects is uniform or non-uniform.

Time	Distance travelled by object P in (m)	Distance travelled by object Q in (m)
9:30 am	10	12
9:45 am	20	19
10:00 am	30	23
10:15 am	40	35
10:30 am	50	37
10:45 am	60	41
11:00 am	70	44

[Delhi Gov. SQP 2022]

Ans. We can see that the object P travels 10 metres every fifteen minutes. In other words, it travels the same distance in the same amount of time. As a result, the motion of item P is uniform. The object Q, on the other hand, covers 7 m from 9:30 to 9:45 a.m., 4 m from 9:45 to 10:00 a.m., and so on. In other words, it travels an uneven distance in an equal amount of time. As a result, the motion of item Q is not uniform.

62. Explain observation clearly with reason: If you look out of the window of a fast moving train, the nearby trees, houses, etc., seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the moon, the stars, etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you). [Delhi Gov. SQP 2022]

Ans. A line of sight is an imaginary line that connects an item with the observer's eye. When we look at surrounding stationary things such as trees, homes, and so on while seated on a moving train, they appear to move quickly in the other direction because the line of sight changes so quickly.

Because of the great distance, distant things such as trees, stars, and so on look fixed. As a result, the line of sight does not switch directions quickly.

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

63. A particle moves along a circular path of radius r. It starts from point A and moves anti-clockwise. Find the distance travelled by the particle as it:

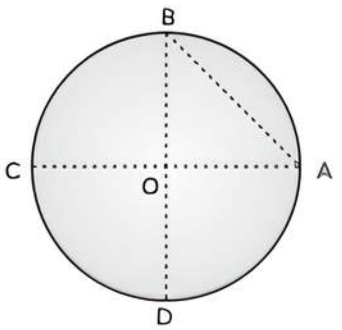
(A) moves from A to B

(B) moves from A to C

(C) moves from A to D

(D) complete one revolution

Find magnitude of distance and displacement in each case.



Ans. (A) From A to B position,

$$\text{Distance Covered} = \frac{1}{4} \times 2\pi r = \frac{1}{2}\pi r$$

Displacement,

$$|\vec{AB}| = \sqrt{OA^2 + OB^2} = \sqrt{r^2 + r^2} = \sqrt{2}r$$

(B) From A to C,

$$\text{Distance covered} = \frac{1}{2} \times 2\pi r = \pi r$$

$$\text{Displacement}, = |\vec{AC}| = 2r$$

(C) From A to D,

$$\text{Distance covered} = \frac{3}{4} \times 2\pi r = \frac{3}{2}\pi r$$

$$\text{Displacement} = |\vec{AD}| = \sqrt{r^2 + r^2} = \sqrt{2}r$$

(D) From A to A,

Distance covered = $2\pi r$

As the final position coincides with the initial position, the displacement covered is zero.

64. A body travelling along a straight line traversed one-half of the total distance with a velocity v_0. The remaining part of the distance was covered with a velocity v_1 for half the time and with velocity v_2 for other half of the time. Find the near velocity averaged over the whole time of motion.

Ans. Let total distance = x

Let the time taken to cover the first one-half distance = t_1

$$\text{Then, } t_1 = \frac{\frac{x}{2}}{v_0} = \frac{x}{2v_0}$$

Let the time taken for the next $\frac{x}{2}$ distance be t_2

$$\text{Then, } \frac{x}{2} = v_1 \cdot \frac{t_2}{2} + v_2 \frac{t_2}{2} = \frac{(v_1 + v_2)}{2} t_2$$

$$t_2 = \frac{x}{v_1 + v_2}$$

Total time taken,

$$t_1 + t_2 = \frac{x}{2v_0} + \frac{x}{v_1 + v_2}$$

$$= \frac{x(v_1 + v_2 + 2v_0)}{2v_0(v_1 + v_2)}$$

$$\text{Average velocity} = \frac{\text{Total distance}}{\text{Total time}}$$

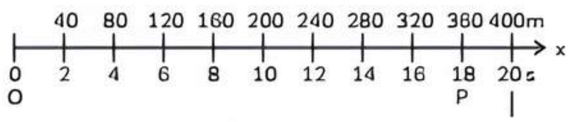
$$= \frac{x}{\frac{x(v_1 + v_2 + 2v_0)}{2v_0(v_1 + v_2)}}$$

$$= \frac{2v_0(v_1 + v_2)}{v_1 + v_2 + 2v_0}$$

65. A car is moving along the x-axis as shown in the figure, it moves from O to P and returns from P to Q. What is the average velocity and average speed of car in going from:

(A) from O to P

(B) from O to P and back to Q



Ans. (A) From O to P

Average velocity

$$= \frac{\text{Displacement}}{\text{Time interval}} = \frac{360}{18 \text{ s}} = 20 \text{ ms}^{-1}$$

(B) From O to P and back to Q

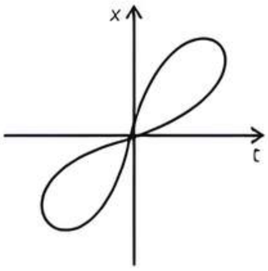
Average velocity

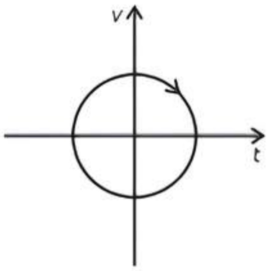
$$= \frac{OQ}{18 + 6} = \frac{240 \text{ m}}{24 \text{ s}} = 10 \text{ ms}^{-1}$$

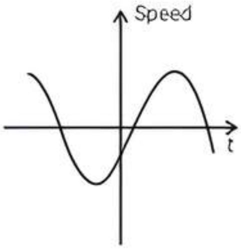
Average speed

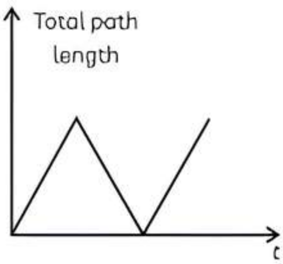
$$= \frac{OP + PQ}{18 + 6} = \frac{360 + 120 \text{ m}}{24 \text{ s}} = 20 \text{ ms}^{-1}$$

66. **A simple one-dimensional consolidation model consists of a rectilinear element of soil subject to vertical changes in loading and through which vertical (only) seepage flow is taking place. According to this definition which of the below graphs are best for explaining one dimensional motion of particles?**

(A) 

(B) 

(C) 

(D) 

Ans. All four graphs are impossible.

(A) If we draw a line parallel to the position-axis, it intersects the x-t graph at two points. This means that the particle occupies two different positions at the same time which is not possible.

(B) If we draw a line parallel to the velocity axis, it meets the circle in two points. This means the particle has two velocities (positive and negative) in opposite directions at the same time. This is also not possible.

(C) The graph indicates that speed is negative in some time intervals. But speed cannot be negative.

(D) The graph indicates that the total path length decreases after a certain time. But total path length can never decrease with time.

67. **The speed of the motor launch with respect to still water is 7 m/s and the speed of the stream is 3 m/s. When the launch began traveling upstream, a float was dropped from it. The launch traveled 4.2 km upstream, turned about, and caught up with the float. How long is it before the launch reaches the floor? [NCERT Exemplar]**

Ans. Speed of the boat when it travels upstream

$= 7 - 3 = 4$ m/sec

Speed of the boat when it travels downstream

$= 7 + 3 = 10$ m/sec

Now,

As soon as the float falls, it will travel downstream at the speed of 3 m/sec.

And in the meantime, the boat has also travelled upstream a distance of 4.2 km.

So,

Time spent on traveling upstream by boat,

$$= \frac{4200}{4} = 1050 \text{ sec}$$

In the meantime, the float would have traveled downstream by, $= 1050 \times 3 = 3150$ m

Hence, the total distance between the float and the boat will be $= 3150 + 4200 = 7350$ m

Now, the relative speed between the boat and the float $= 10 - 3 = 4$ m/sec

Hence, time taken $= \frac{7350}{4} = 1837.5$ sec

Hence, total time spent

$= 1837.5 + 1050$

$= 2887.5$ sec

This is $\frac{2887.5}{60} = 48.125$ minutes

Hence, the float and the boat will meet in 48.125 minutes.

68. **An electron travelling with a speed of 5×10^3 m/s passes through an electric field with an acceleration of 10^{12} m/s^2. How long will it take for the electron to double its speed? What will be the distance covered by the electron at this time? [NCERT Exemplar]**

Ans. Here, $u = 5 \times 10^3$ m/s, $v = 2 \times 5 \times 10$ m/s,

$a = 10^{12}$ m/s^2

(A) Time(t) $= 2 \times 5 \times 10 - \frac{5 \times 10^3}{10^{12}} = 5 \times 10^{-9}$ s

(B) Distance, $s = ut + \frac{1}{2}at^2 = 5 \times 10^3 \times 5 \times 10^{-9}$

$$+\frac{1}{2}(10^{12} \times 5 \times 10^{-9} \times 5 \times 10^{-9})$$

$$= 3.75 \times 10^{-5} m$$

69. **Air distance Kota to Jaipur is 260 km and road distance is 320 km a deluxe bus which moves from Jaipur to Kota takes 8 hours while an airplane reaches in just 15 min. Find:**
(A) Average speed of the bus in km/h.
(B) Average velocity of the bus in km/h.
(C) Average velocity of an airplane in km/h.
(D) Average velocity of airplane in km/h.

Ans. Air distance (the shortest distance)
Displacement = 260 km
Road distance = 320 km

(A) Average speed of bus,

$$\frac{\text{Total distance}}{\text{Total time}} = \frac{320}{8} = 40 \text{ km/h}$$

(B) Average velocity of bus,

$$\frac{\text{Total displacement}}{\text{Total time}} = \frac{260}{8} = 32.5 \text{ km/h}$$

(C) Average speed of plane,

$$= \frac{260}{\frac{1}{4}} = 1040 \text{ km/h}$$

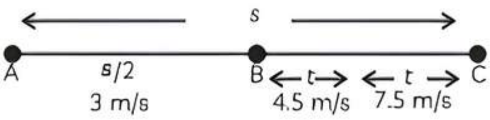
(D) Average velocity of airplane,

$$= \frac{260}{\frac{1}{4}} = 1040 \text{ km/h}$$

NUMERICAL Type Questions

70. **A particle moves on a straight line in such a way that it covers 1st half distance with speed 3 m/s and the next half distance in 2 equal time intervals with speeds 4.5 m/s and 7.5 m/s respectively. Find the average speed of particles.** **(2m)**

Ans.



Average speed from B to C,

$$(v_{avg})_{B\to C} = \frac{4.5 \times t + 7.5 \times t}{2t}$$

$$(v_{avg})_{B\to C} = 6 \text{ m/s}$$

Average speed from A to C,

$$(v_{avg})_{A\to C} = \frac{\frac{s}{2} + \frac{s}{2}}{\frac{s}{2\times 3} + \frac{s}{2\times 6}}$$

$$= 4 \text{ m/s}$$

71. **A particle of mass 2 kg moves on a circular path with a constant speed of 10 m/s. Find the change in speed and magnitude of change in velocity. When a particle completes one revolution.** **(2m)**

Ans. Change in speed

$$\Delta v = 10 - 10 = 0$$

Changes in velocity

$$\Delta \vec{v} = \vec{v}_1 - \vec{v}_1$$

$$= 10\hat{i} - (-10\hat{i}) = 20\hat{i}$$

Magnitude, $\Delta \vec{v} = 20$ m/s

72. **A particle goes from point A to point B, moving in a semicircle of radius 1 m in 1 second. Find the magnitude of its average velocity.** **(2m)**

Ans. Average velocity

$$= \frac{\text{Net displacement}}{\text{Total time}}$$

$$= \frac{AO + OB}{\text{time}}$$

$$= \frac{1+1}{1} = 2 \text{ m/s}$$

73. **The straight distance between a hotel and a railway station is 10 km but the circular route is followed by a train covering 23 km in 28 minutes.**
What is the average speed and magnitude of average velocity? Are they equal? **(2m)**

Ans. (A) Average speed $= \frac{\text{Total path length}}{\text{Total time}}$

$$= \frac{23}{28/60} = 49.3 \text{ kmh}^{-1}$$

(B) Average velocity,

$$= \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{\text{Shortest distance}}{\text{Time}}$$

$$= \frac{10}{28/60} = 21.4 \text{ kmh}^{-1}$$

(C) Both are not equal.

74. **The velocity-time relation of an electron starting from rest is given by $v = kt$, where $k = 2\ ms^{-2}$. Calculate the distance traversed in 3 s. (2m)**

Ans. Velocity, $v = kt$

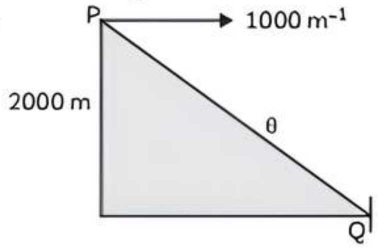
Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(kt) = k = 2\ ms^{-2}$

Distance traversed in 3 s,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times (3)^2 = 9\ m$$

75. **An airplane is travelling horizontally at a height of 2000 m from the ground. The airplane when at a point P drops a bomb to hit a stationary target Q on the ground. In order to hit the target, at what angle θ must the line PQ make with the vertical? (2m)**

Ans.



Let t be the time taken by the bomb to hit the target.

$$h = 2000\ m,\ h = \frac{1}{2}gt^2.$$

so, $t = 2000\ m$

$$R = ut$$
$$= (100)(20)$$
$$= 2000\ m$$

$$\tan\theta = \frac{R}{h} = \frac{2000}{2000} = 1$$

So, $\theta = 45°$

76. **A ball rolls off the top of a stairway with a horizontal velocity u. If each step has height h and width b the ball will just hit the edge of the n^{th} step. Find the value of n. (2m)**

Ans. If the ball hits the n^{th} step, the horizontal and vertical distances traversed are n_b and n_h respectively. Let t be the time taken by the ball for these horizontal and vertical displacements. Velocity along horizontal direction = u (remains constant) and the initial velocity is zero.

$$n_b = ut \text{ and } n_h = 0 + \frac{1}{2}gt^2$$

Eliminating t from the equation,

$$n_h = \frac{1}{2}g\left(\frac{nb}{u}\right)^2$$

$$n = \frac{2hu^2}{gb^2}$$

77. **The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. Calculate the acceleration.**
[NCERT Exemplar](2m)

Ans. Given that, $t = ax^2 + bx$

Differentiate w.r.t time,

$$1 = 2ax\frac{dx}{dt} + b\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{2ax + b}$$

$$v = \frac{1}{2ax + b} \quad \text{...(i)}$$

$$a = \frac{dv}{dt} = \frac{-2a\left(\frac{dx}{dt}\right)}{(2ax + b)^2}$$

$$a = \frac{-2av}{(2ax + b)^2} \quad \text{...(ii)}$$

Substituting, $\frac{1}{2ax + b} = v$ in eqn. (ii)

we will get, $a = -2av^3$.

78. **The driver of a train moving at a speed v_1 observes another train at a distance d ahead of him on the same track moving in the same direction with a slower speed v_2. He applies the brakes and gives his train constant deceleration α. Show that if $d > \frac{(v_1 - v_2)^2}{2\alpha}$, there will be no collision and if $d < \frac{(v_1 - v_2)^2}{2\alpha}$, there will be a collision. (2m)**

Ans. There will be no collision if the driver of the train moving at a speed v_1 reduces the speed of his train to v_2 before the two trains meet, *i.e.*, the relative velocity of the two trains must be reduced to zero.

Initial relative velocity = $v_1 - v_2$

If s is the distance covered by the first train before its relative velocity becomes zero, then

$$v^2 - u^2 = 2as$$

or $$\frac{v^2 - u^2}{2\alpha} = s$$

For no collision, $d > s$

So, $$d < \frac{(v^2 - v^2)^2}{2\alpha}$$

79. **A parachutist after bailing outfalls 50 m without friction. When the parachute opens, it decelerates at 2 ms^{-2}. He reaches the ground with a speed of 3 ms^{-2}. At what height, did he bail out? (2m)**

Ans. Velocity attained after falling freely through first 50 *m*.

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 50} = \sqrt{980}\ \text{m/s}$$

After the parachute opens.

$$u = \sqrt{980}\ \text{m/s}; \quad v = 3\ \text{m/s}; \quad a = -2\ \text{ms}^{-2}$$

$$v^2 - u^2 = 2gh$$

$$9 - 980 = 2 \times 2 \times h$$

or $$h = 243\ \text{m.}$$

80. Tom and Jack are running forward with the same speed. They are following a rubber ball to each other at a constant speed *v* as seen by the thrower. Calculate the speed of the ball according to Sam, who is standing on the ground. (2m)

Ans. As both Tom and Jack are moving in the same direction, the relative velocity of the ball with respect to Tom or Jack will be;

$$v = v_b - v_{Tom}$$

$$v = v_b - v_{Jack}$$

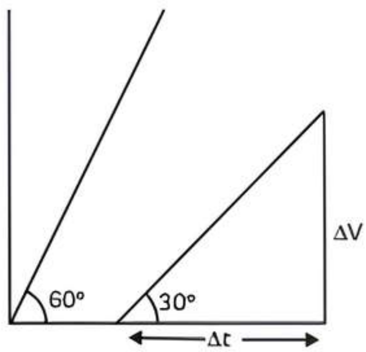
$$v_b = v + v_{Tom} = v + v_{Jack}$$

For Sam, who is standing on the ground, the speed of the ball will be greater than *v*.

81. The *v*–*t* graphs of two objects make angle 30° and 60° with the time axis. Find the ratio of their accelerations.

[Delhi Gov. QB 2022](3m)

Ans.



$$\tan\theta = \frac{\Delta V}{\Delta t} = a$$

$$a = \frac{\Delta V}{\Delta t} = \text{slope of } v\text{–}t \text{ graph}$$

$$a_1 = \tan 30° = \frac{1}{\sqrt{3}}$$

$$a_2 = \tan 60° = \sqrt{3}$$

$$\frac{a_1}{a_2} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3} \times \sqrt{3}} = \frac{1}{3}$$

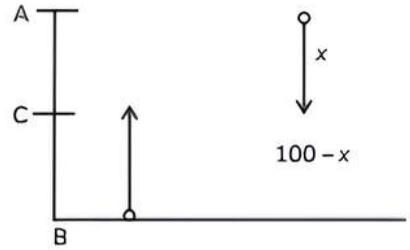
82. From the top of a tower 100 m in height a ball is dropped and at the same time another ball is projected vertically upwards from the ground with a velocity of 25 m/s. Find when and where the two balls will meet. (g = 9.8 m/s) [Delhi Gov. QB 2022](3m)

Ans. Let, $$AC = x$$

Then, $$BC = 100 - x$$

$$x = ut + \frac{1}{2}gt^2$$

$$x = 4.9\,t^2 \quad \text{...(i)}$$



$$BC = ut - \frac{1}{2}gt^2$$

$$= 25t - \frac{1}{2} \times 9.8t^2$$

$$100 - x = 25t - 4.9\,t^2 \quad \text{...(ii)}$$

Substituting, $4.9t^2 = x$ from eq (i) in (ii).

$$100 - 2 = 25t - x$$

$\Rightarrow$ $$t = 4 \text{ seconds}$$

$$x = 4.9\,t^2$$

$$= 4.9 \times 4^2$$

= 78.4 m from the top or 21.6 m from the ground.

83. Two town A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed do the busses ply of the road? [Delhi Gov. QB 2020](3m)

Ans. Let V be the speed of the bus running between towns A and B.

Speed of the cyclist, v = 20 km/h

Relative speed of the bus moving in the direction of the cyclist = V – v = (V – 20) km/h

The bus went past the cyclist every 18 min *i.e.*, $\frac{18}{60}h$

(when he moves in the direction of the bus).

Distance covered by the bus

$$= (V - 20)\frac{18}{60}\ \text{km} \quad \text{...(i)}$$

Since one bus leaves after every T minutes, the distance travelled by the bus will be equal to

$$V \times \frac{T}{60} \quad ...(ii)$$

Both equations (i) and (ii) are equal.

$$(V-20) \times \frac{18}{60} = \frac{VT}{60} \quad ...(iii)$$

Relative speed of the bus moving in the opposite direction of the cyclist = (V + 20) km/h

Time taken by the bus to go past the cyclist

$$= 6 \text{ min} = \frac{6}{60}\text{h}$$

$$\therefore \quad (V+20)\frac{6}{60} = \frac{VT}{60} \quad ...(iv)$$

From equations (iii) and (iv), we get

$$(V+20) \times \frac{6}{60} = (V-20) \times \frac{18}{60}$$

$$V + 20 = 3V - 60$$

$$2V = 80$$

$$V = 40 \text{ km/h}$$

Substituting the value of V in equation (iv), we get

$$(40+20) \times \frac{6}{60} = \frac{40T}{60}$$

$$T = \frac{360}{40}$$

$$= 9 \text{ min}$$

84. A driver take 0.20 second to apply the breaks (reaction time). If he is driving car at a speed of 54 kmh^{-1} and the breaks cause a deceleration of 6.0 ms^{-2}. Find the distance travelled by car after he sees the need to put the breaks. [Delhi Gov. QB 2022](3m)

Ans. Given: Then initial speed of the car is 54 km/hr or 15 m/s.

In reaction time the body moves with the speed 54 km/hr = 15 m/s (constant speed)

Distance travelled in this time is

$$S_1 = u \times t$$

$$S_1 = 15 \times 0.2$$

$$= 3 \text{ m}$$

When brakes are applied.

u = 15 m/s, v = 0, a = – 6 m/s^2 (deceleration)

$$v^2 = u^2 + 2aS_2$$

$$S_2 = \frac{v^2 - u^2}{2a}$$

$$= \frac{0 - 15^2}{2(-6)}$$

$$= 18.75 \text{ m}$$

Total distance $S = S_1 + S_2$

$$= 3 + 18.75$$

$$= 21.75$$

$$= 22 \text{ m}$$